GRE practice test 2008.
E&M and optics.

number	Category	
3	E&M	E
4	E&M	E
15	optics and waves	E
16	optics and waves	D
17	E&M	A
18	E&M	E
36	E&M	A
37	E&M	E
38	E&M	E
45	optics and waves	C
46	optics and waves	D
47	optics and waves	D
52	optics and waves	C
60	E&M	C
61	E&M	E
62	E&M	E
67	E&M	D
68	E&M	D
69	E&M	D
70	E&M	A
71	E&M	B
74	optics and waves	E
75	optics and waves	B
76	optics and waves	B
90	E&M	D
91	E&M	C
92	E&M	E
93	E&M	B

answer key and explanation:

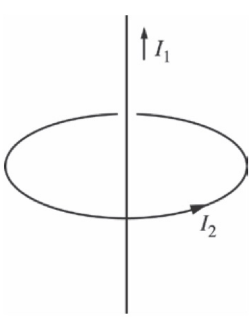
3. A resistor in a circuit dissipates energy at a rate of 1 W. If the voltage across the resistor is doubled, what will be the new rate of energy dissipation?

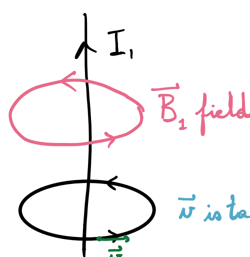
(A) 0.25 W
(B) 0.5 W
(C) 1 W
(D) 2 W
(E) 4 W

$P_{resistor} = I|\Delta V| \qquad \Delta V_R = -IR$

$= \frac{|\Delta V|^2}{R} = I^2 R$

$P_2 = \frac{|\Delta V_2|^2}{R} = \frac{|2\Delta V_1|^2}{R} = 4\,\frac{|\Delta V_1|^2}{R} = 4P_1$





$\vec F_B = q\vec v \times \vec B$

$\vec v$ is tangent to the circle

at all point in loop 2, $\vec B_1 // \vec v_2$

$\Rightarrow \vec v_2 \times \vec B_1 = \vec 0 \Rightarrow \vec F_B = \vec 0$

4. An infinitely long, straight wire carrying current I_1 passes through the center of a circular loop of wire carrying current I_2, as shown above. The long wire is perpendicular to the plane of the loop. Which of the following describes the magnetic force on the loop?

(A) Outward, along a radius of the loop.
(B) Inward, along a radius of the loop.
(C) Upward, along the axis of the loop.
(D) Downward, along the axis of the loop.
(E) There is no magnetic force on the loop.

15. If the five lenses shown below are made of the same material, which lens has the shortest positive focal length?

(A)

(B)

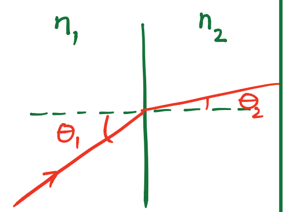
(C)

(D)

(E)

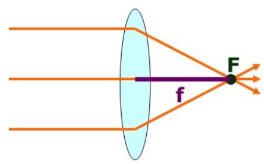
Snell law:

$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

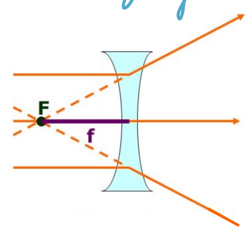


$\left(\begin{array}{l} n_1 < n_2 \Rightarrow \theta_1 > \theta_2 \\ n_1 > n_2 \Rightarrow \theta_1 < \theta_2 \end{array} \right)$

converging lenses:



diverging lenses:



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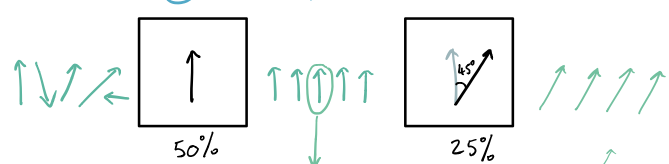
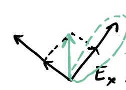
16. Unpolarized light is incident on a pair of ideal linear polarizers whose transmission axes make an angle of 45° with each other. The transmitted light intensity through both polarizers is what percentage of the incident intensity?

(A) 100%
(B) 75%
(C) 50%
(D) 25%
(E) 0%

polarizer select one particular direction for the $\vec{E}$ field.

natural light: unpolarized: ↑ ↓ ↖ ↗ ↑ ↓





$E_x = E_y = E\cos(45) = \frac{E}{\sqrt{2}}$

17. A very long, thin, straight wire carries a uniform charge density of λ per unit length. Which of the following gives the magnitude of the electric field at a radial distance r from the wire?

(A) $\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$

(B) $\frac{1}{2\pi\varepsilon_0}\frac{r}{\lambda}$

(C) $\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r^2}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda^2}{r^2}$

(E) $\frac{1}{4\pi\varepsilon_0}\lambda \ln r$

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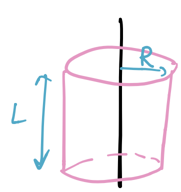
1) dimensional analysis: $E = k\frac{q}{r^2}$

$q \propto \lambda [\ell]$ $\quad \left(V = k\frac{q}{r}\right)$

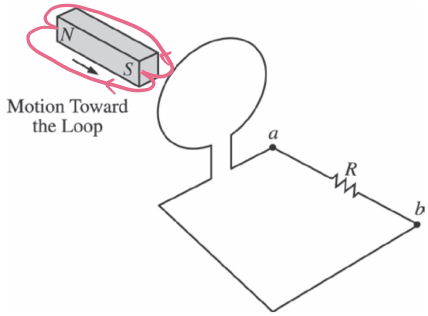
$\Rightarrow E \propto \frac{\lambda}{[\ell]}$

2) Gauss law

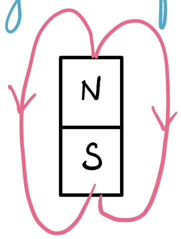


$E\, 2\pi R L = \frac{\lambda L}{\varepsilon_0}$

$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 R}$



magnetic field created by a magnet:



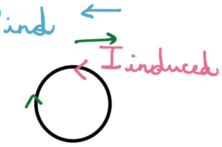
18. The bar magnet shown in the figure above is moved completely through the loop. Which of the following is a true statement about the direction of the current flow between the two points a and b in the circuit?

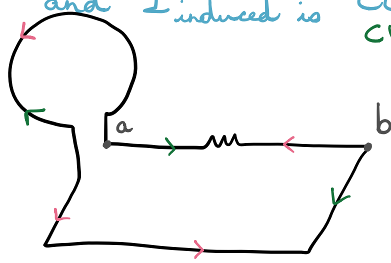
(A) No current flows between a and b as the magnet passes through the loop.
(B) Current flows from a to b as the magnet passes through the loop.
(C) Current flows from b to a as the magnet passes through the loop.
(D) Current flows from a to b as the magnet enters the loop and from b to a as the magnet leaves the loop.
(E) Current flows from b to a as the magnet enters the loop and from a to b as the magnet leaves the loop.

$\longrightarrow$: $\Phi_B \nearrow$ and $\vec{B}$ is $\rightarrow$ $\Rightarrow$ $\vec{B}_{ind}$ $\leftarrow$

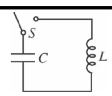
$\longleftarrow$: $\Phi_B \searrow$ and $\vec{B}$ is $\rightarrow$ $\Rightarrow$ $\vec{B}_{ind}$ $\rightarrow$

and $I_{induced}$ is CCW
CW

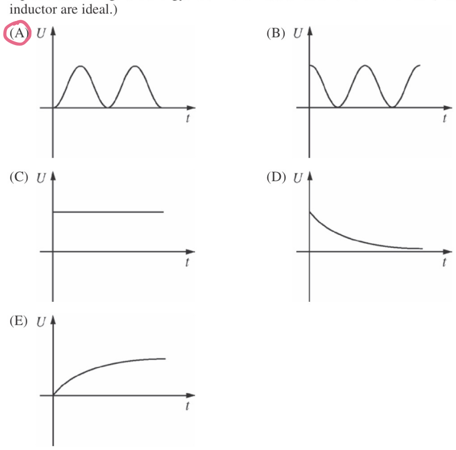




$$\boxed{\text{Lenz law: } \quad \mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \Phi_B = \iint \vec{B}\cdot d\vec{A}}$$



36. The capacitor in the circuit above is charged. If switch S is closed at time $t = 0$, which of the following represents the magnetic energy, U, in the inductor as a function of time? (Assume that the capacitor and inductor are ideal.)



for an LC circuit $I(t) = I_0 \cos(\omega t + \phi)$

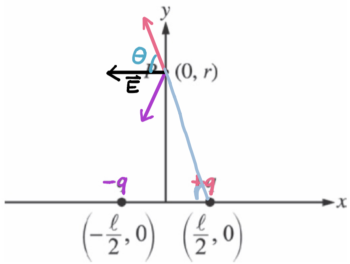
$$\omega = \frac{1}{\sqrt{LC}}$$

and $U_L = \frac{1}{2} L I^2$

$$\int_{-\infty}^{+\infty} dx\, e^{-x^2} = \sqrt{\pi}$$

hence A

$$L\frac{d^2q}{dt^2} + \frac{1}{C} q = 0.$$



- because of the geometry of the setup, $\vec{E}$ is in the $-\hat{x}$ direction.
- $|\vec{E}|$ has to depend on ℓ.
- $|\vec{E}| \propto \frac{q}{r^2}$

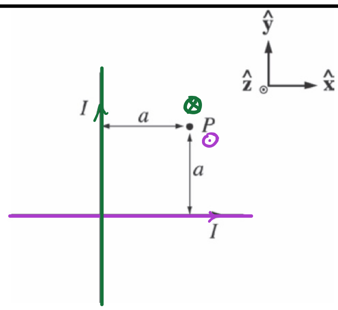
37. A pair of electric charges of equal magnitude q and opposite sign are separated by a distance ℓ, as shown in the figure above. Which of the following gives the approximate magnitude and direction of the electric field set up by the two charges at a point P on the y-axis, which is located a distance $r >> \ell$ from the x-axis?

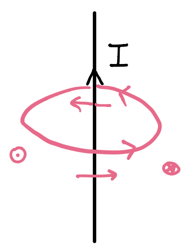
$$E_+ = E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+\frac{\ell^2}{4}}$$

$$E = 2E_+\cos\theta = \frac{1}{2\pi\varepsilon_0}\frac{q}{r^2+\frac{\ell^2}{4}} \times \frac{\ell/2}{\left(r^2+\frac{\ell^2}{4}\right)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\ell q}{\left(r^2+\frac{\ell^2}{4}\right)^{3/2}} \xrightarrow[r\gg\ell]{} \frac{1}{4\pi\varepsilon_0}\frac{\ell q}{r^3}$$

	Magnitude	Direction
(A)	$\frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$	$+y$
(B)	$\frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$	$+x$
(C)	$\frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$	$-x$
(D)	$\frac{1}{4\pi\epsilon_0}\frac{q\ell}{r^3}$	$+x$
(E)	$\frac{1}{4\pi\epsilon_0}\frac{q\ell}{r^3}$	$-x$





38. Consider two very long, straight, insulated wires oriented at right angles. The wires carry currents of equal magnitude I in the directions shown in the figure above. What is the net magnetic field at point P ?

$$|\vec{B}| = |\vec{B}| = \frac{\mu_0}{2\pi}\frac{I}{a} \Rightarrow \vec{B}_{tot} = \vec{0}$$

(A) $\frac{\mu_0 I}{2\pi a}(\hat{\mathbf{x}}+\hat{\mathbf{y}})$

(B) $-\frac{\mu_0 I}{2\pi a}(\hat{\mathbf{x}}+\hat{\mathbf{y}})$

(C) $\frac{\mu_0 I}{\pi a}\hat{\mathbf{z}}$

(D) $-\frac{\mu_0 I}{\pi a}\hat{\mathbf{z}}$

(E) $\mathbf{0}$

45. During a hurricane, a 1,200 Hz warning siren on the town hall sounds. The wind is blowing at 55 m/s in a direction from the siren toward a person 1 km away. With what frequency does the sound wave reach the person? (The speed of sound in air is 330 m/s.)

(A) 1,000 Hz
(B) 1,030 Hz
(C) 1,200 Hz
(D) 1,400 Hz
(E) 1,440 Hz

Not a Doppler effect pb!
⇒ the source and the observer are both stationary!!!

$$\boxed{\text{Doppler effect: } f_o = \frac{v \pm v_o}{v \pm v_s} f_s}$$

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46. Sound waves moving at 350 m/s diffract out of a speaker enclosure with an opening that is a long rectangular slit 0.14 m across. At about what frequency will the sound first disappear at an angle of 45° from the normal to the speaker face?

(A) 500 Hz
(B) 1,750 Hz
(C) 2,750 Hz
(D) 3,500 Hz
(E) 5,000 Hz

diffraction : (= single slit)

constructive : $\Delta x = (n+\frac{1}{2})\lambda$

destructive : $\Delta x = n\lambda$.

here we are looking at destructive

$\lambda_n = \frac{\Delta x}{n}$ and $\Delta x = d\sin(45)$

$\Rightarrow \lambda_n = \frac{1}{n}\frac{d}{\sqrt{2}} \Rightarrow f_n = \frac{\sqrt{2}\,n}{d} v$

$f_1 = \frac{\sqrt{2}}{0.14} \times 350 = 3536\,\text{Hz}$

47. An organ pipe, closed at one end and open at the other, is designed to have a fundamental frequency of C (131 Hz). What is the frequency of the next higher harmonic for this pipe?

(A) 44 Hz
(B) 196 Hz
(C) 262 Hz
(D) 393 Hz
(E) 524 Hz

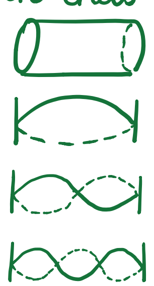
one end closed one end open:

$$\lambda_{2n-1} = \frac{4L}{2n-1} = \frac{\lambda_1}{2n-1} \Rightarrow f_{2n-1} = \frac{(2n-1)v}{4L} = (2n-1)f_1$$

here $f_1 = 131$ Hz, the next harmonic is $n=2$ (one end closed, one end open)

$$f_3 = 3 \times f_1 = 393\text{ Hz}$$

• both ends fixed



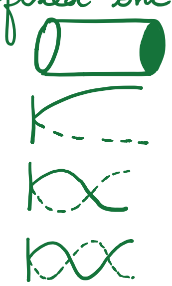
$\lambda_1 = 2L$

$\lambda_2 = L$

$\lambda_3 = \frac{2}{3}L$

$$\boxed{\lambda_n = \frac{2L}{n} = \frac{\lambda_1}{n} \qquad f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1}$$

• one end fixed one end open



$\lambda_1 = 4L$

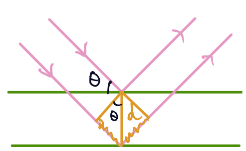
$\lambda_3 = \frac{4L}{3}$

$\lambda_5 = \frac{4L}{5}$

$$\boxed{\lambda_{2n-1} = \frac{4L}{2n-1} = \frac{\lambda_1}{2n-1} \qquad f_{2n-1} = \frac{v}{\lambda_{2n-1}} = \frac{(2n-1)v}{4L} = (2n-1)f_1}$$

52. X rays of wavelength $\lambda = 0.250$ nm are incident on the face of a crystal at angle θ, measured from the crystal surface. The smallest angle that yields an intense reflected beam is $\theta = 14.5°$. Which of the following gives the value of the interplanar spacing d? ($\sin 14.5° \approx 1/4$)

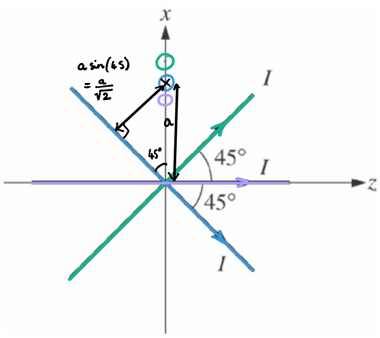
(A) 0.125 nm
(B) 0.250 nm
(C) 0.500 nm
(D) 0.625 nm
(E) 0.750 nm



Bragg reflection:

$$\Delta x = 2d\sin\theta = \frac{d}{2} = \lambda$$

$$\Rightarrow \underline{d = 2\lambda}$$



$$\left|\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{I}{a} = \left|\vec{B}_2\right|$$

$$\left|\vec{B}_1\right| = \left|\vec{B}_3\right| = \frac{\mu_0}{4\pi}\frac{I}{a/\sqrt{2}}$$

$$\left|\vec{B}_{tot}\right| = \left|\vec{B}_1\right| + \left|\vec{B}_2\right| + \left|\vec{B}_3\right| = \frac{\mu_0}{4\pi}\frac{I}{a}\left(1+2\sqrt{2}\right)$$

60. Three long, straight wires in the xz-plane, each carrying current I, cross at the origin of coordinates, as shown in the figure above. Let $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ denote the unit vectors in the x-, y-, and z-directions, respectively. The magnetic field $\mathbf{B}$ as a function of x, with $y = 0$ and $z = 0$, is

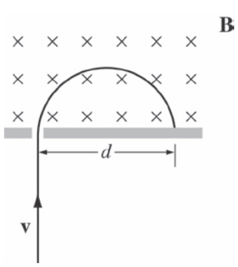
(A) $\mathbf{B} = \frac{3\mu_0 I}{2\pi x}\hat{\mathbf{x}}$

(B) $\mathbf{B} = \frac{3\mu_0 I}{2\pi x}\hat{\mathbf{y}}$

(C) $\mathbf{B} = \frac{\mu_0 I}{2\pi x}\left(1+2\sqrt{2}\right)\hat{\mathbf{y}}$

(D) $\mathbf{B} = \frac{\mu_0 I}{2\pi x}\hat{\mathbf{x}}$

(E) $\mathbf{B} = \frac{\mu_0 I}{2\pi x}\hat{\mathbf{y}}$



once in region with uniform $\vec{B}$

$\Rightarrow$ circular motion

$$a = \frac{v^2}{R}$$

Newton's second law:

$$ma = F_B \Rightarrow m\frac{v^2}{R} = qvB$$

$\Rightarrow R = \frac{m}{q}\frac{v}{B}$ hence if $\frac{q}{m}$ is doubled R is half.

61. A particle with mass m and charge q, moving with a velocity $\mathbf{v}$, enters a region of uniform magnetic field $\mathbf{B}$, as shown in the figure above. The particle strikes the wall at a distance d from the entrance slit. If the particle's velocity stays the same but its charge-to-mass ratio is doubled, at what distance from the entrance slit will the particle strike the wall?

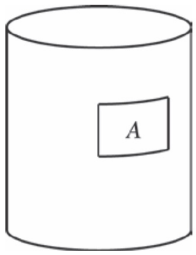
(A) $2d$

(B) $\sqrt{2}d$

(C) d

(D) $\frac{1}{\sqrt{2}}d$

(E) $\frac{1}{2}d$



Gauss law:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{ins}}{\varepsilon_0}$$

here the total flux has to be positive because the charge inside the surface is positive

↳ the only choice is (E)

62. Consider the closed cylindrical Gaussian surface above. Suppose that the net charge enclosed within this surface is $+1 \times 10^{-9}$ C and the electric flux out through the portion of the surface marked A is $-100\ \text{N}\cdot\text{m}^2/\text{C}$. The flux through the rest of the surface is most nearly given by which of the following?

(A) $-100\ \text{N}\cdot\text{m}^2/\text{C}$
(B) $0\ \text{N}\cdot\text{m}^2/\text{C}$
(C) $10\ \text{N}\cdot\text{m}^2/\text{C}$
(D) $100\ \text{N}\cdot\text{m}^2/\text{C}$
(E) $200\ \text{N}\cdot\text{m}^2/\text{C}$

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67. A large, parallel-plate capacitor consists of two square plates that measure 0.5 m on each side. A charging current of 9 A is applied to the capacitor. Which of the following gives the approximate rate of change of the electric field between the plates?

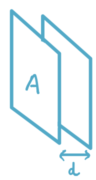
(A) $2\ \frac{\text{V}}{\text{m}\cdot\text{s}}$

(B) $40\ \frac{\text{V}}{\text{m}\cdot\text{s}}$

(C) $1 \times 10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}$

(D) $4 \times 10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}$

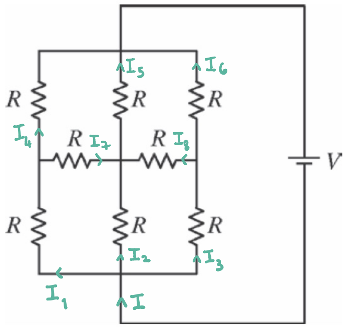
(E) $2 \times 10^{13}\ \frac{\text{V}}{\text{m}\cdot\text{s}}$



$$C = \frac{\varepsilon A}{d} \Rightarrow V_c = \frac{Q}{C} = \frac{Qd}{\varepsilon A}$$

$$E = \frac{V}{d} \Rightarrow \frac{dE}{dt} = \frac{dQ}{dt} \times \frac{1}{\varepsilon A} = \frac{I}{\varepsilon A}$$

$$\simeq 4\times10^{12}\,\frac{V}{ms}$$



68. The circuit shown in the figure above consists of eight resistors, each with resistance R, and a battery with terminal voltage V and negligible internal resistance. What is the current flowing through the battery?

(A) $\frac{1}{3}\frac{V}{R}$

(B) $\frac{1}{2}\frac{V}{R}$

(C) $\frac{V}{R}$

(D) $\frac{3}{2}\frac{V}{R}$

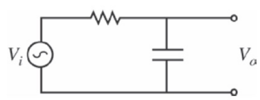
(E) $3\frac{V}{R}$

$$\begin{cases} V = I_1 R + I_4 R \\ V = I_2 R + I_5 R \\ V = I_3 R + I_6 R \\ I_1 + I_2 + I_3 = I \\ I_4 + I_5 + I_6 = I \\ I_1 = I_4 + I_7 \\ I_2 + I_8 + I_7 = I_5 \\ I_3 = I_6 + I_8 \end{cases}$$

$3V = (I_1 + I_2 + I_3)R + (I_4 + I_5 + I_6)R$

$3V = 2IR$

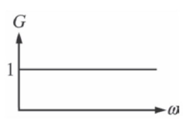
$\Rightarrow I = \frac{3}{2}\frac{V}{R}$

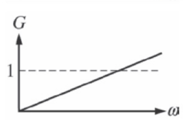


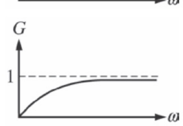
V_i is like a batterie

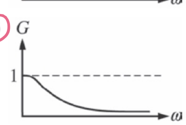
V_o exponential decay

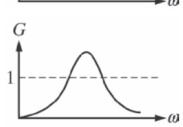
69. In the AC circuit above, V_i is the amplitude of the input voltage and V_o is the amplitude of the output voltage. If the angular frequency ω of the input voltage is varied, which of the following gives the ratio $V_o/V_i = G$ as a function of ω?

(A) 

(B) 

(C) 

(D) 

(E) 

$V_i - IR + V_o = 0$

$V_o = IR - V_i = IR - \mathcal{E}\cos(\omega t)$

$\frac{Q}{C} = \frac{dQ}{dt}R - \mathcal{E}\cos(\omega t)$

$\frac{dQ}{dt} = \frac{Q}{RC} + \mathcal{E}\cos(\omega t)$

low pass filter.

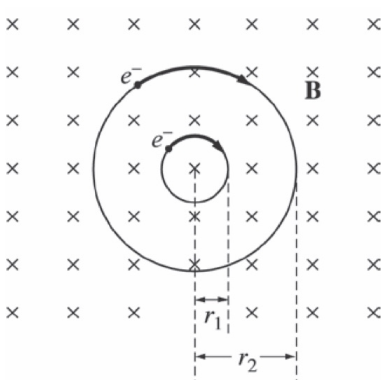
70. A wire loop that encloses an area of 10 cm^2 has a resistance of $5\,\Omega$. The loop is placed in a magnetic field of 0.5 T with its plane perpendicular to the field. The loop is suddenly removed from the field. How much charge flows past a given point in the wire?

(A) 10^{-4} C
(B) 10^{-3} C
(C) 10^{-2} C
(D) 10^{-1} C
(E) 1 C

Induction problem: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

here $\Delta\Phi_B = BA = 0.5T \times 10 \times 10^{-4} m^2 = 5\times10^{-4} T.m^2$

$I = \frac{\mathcal{E}}{R} = \frac{5\times10^{-4}}{5} = 10^{-4} A$



71. Two nonrelativistic electrons move in circles under the influence of a uniform magnetic field **B**, as shown in the figure above. The ratio r_1/r_2 of the orbital radii is equal to 1/3. Which of the following is equal to the ratio v_1/v_2 of the speeds?

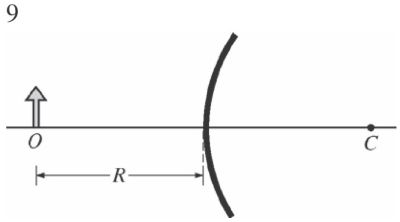
(A) 1/9
(B) 1/3
(C) 1
(D) 3
(E) 9

circular motion $\Rightarrow a = \frac{v^2}{R}$

• $m_e \frac{v_1^2}{r_1^2} = q_e v_1 B \Rightarrow \frac{v_1}{r_1} = q_e B/m$

• $m_e \frac{v_2^2}{r_2^2} = q_e v_2 B \Rightarrow \frac{v_2}{r_2} = q_e B/m$

$\frac{v_1}{r_1} = \frac{v_2}{r_2} \Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{1}{3}$



74. The figure above shows an object O placed at a distance R to the left of a convex spherical mirror that has a radius of curvature R. Point C is the center of curvature of the mirror. The image formed by the mirror is at

(A) infinity
(B) a distance R to the left of the mirror and inverted
(C) a distance R to the right of the mirror and upright
(D) a distance $\frac{R}{3}$ to the left of the mirror and inverted
(E) a distance $\frac{R}{3}$ to the right of the mirror and upright

$o = R = 2|f|$ here convex mirror $\Rightarrow f < 0$

$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{2|f|} + \frac{1}{i} = -\frac{1}{|f|}$

$\Rightarrow i = -\frac{2|f|}{3} = -\frac{R}{3}$ (virtual image)

optics: lenses and mirror

$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$ $\quad M = -\frac{i}{o} = \frac{h_i}{h_o}$

75. A uniform thin film of soapy water with index of refraction $n = 1.33$ is viewed in air via reflected light. The film appears dark for long wavelengths and first appears bright for $\lambda = 540$ nm. What is the next shorter wavelength at which the film will appear bright on reflection?

(A) 135 nm
(B) 180 nm
(C) 270 nm
(D) 320 nm
(E) 405 nm

thin films :
- from low n to high n
  → no phase difference
- from high n to low n
  → phase difference of π.

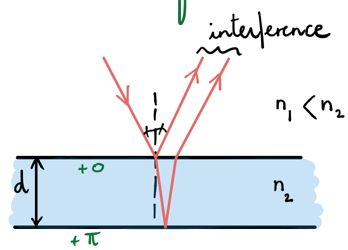
Interference problem.
λ appears bright means constructive interference.

$$\left(n+\frac{1}{2}\right)\lambda = \Delta x = 2d$$

$$\frac{1}{2}\lambda_1 = 2d \qquad \lambda_1 = 540\,\text{nm}$$

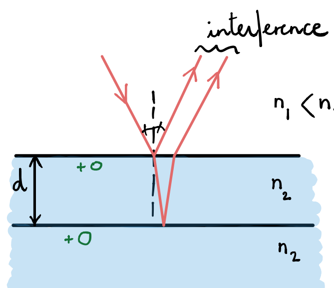
$$\frac{3}{2}\lambda_2 = 2d \Rightarrow \lambda_2 = \frac{2}{3}\times 2d = \frac{2}{3}\times\frac{1}{2}\lambda_1 = \frac{\lambda_1}{3} = 180\,\text{nm}$$

thin films :



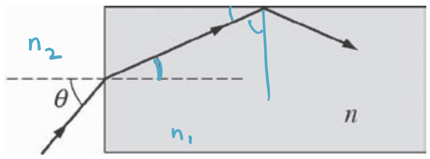
constructive interference : $\Delta x = \left(n+\frac{1}{2}\right)\lambda$

destructive interferences : $\Delta x = n\lambda$



constructive interference : $\Delta x = n\lambda$

destructive interferences : $\Delta x = \left(n+\frac{1}{2}\right)\lambda$



76. A model of an optical fiber is shown in the figure above. The optical fiber has an index of refraction, n, and is surrounded by free space. What angles of incidence, θ, will result in the light staying in the optical fiber?

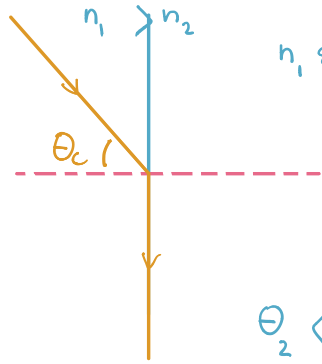
(A) $\theta > \sin^{-1}\left(\sqrt{n^2-1}\right)$

(B) $\theta < \sin^{-1}\left(\sqrt{n^2-1}\right)$

(C) $\theta > \sin^{-1}\left(\sqrt{n^2+1}\right)$

(D) $\theta < \sin^{-1}\left(\sqrt{n^2+1}\right)$

(E) $\sin^{-1}\left(\sqrt{n^2-1}\right) < \theta < \sin^{-1}\left(\sqrt{n^2+1}\right)$



$n_1 \sin\theta_c = \overset{1}{n_2} \sin 90$

$\theta_c = \arcsin\left(\frac{1}{n}\right)$

$\theta > \theta_c$

$\theta_2 < 90 - \theta_c$

$n_2 \sin\theta = n \sin(\theta_2) \Rightarrow \theta = \arcsin\left(n \sin\theta_2\right)$

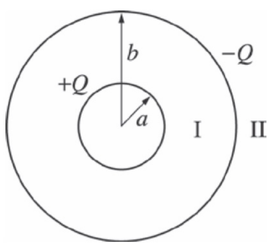
$\theta < \arcsin\left(n\, \underbrace{\sin\left(90 - \arcsin\left(\frac{1}{n}\right)\right)}\right)$

$n\cos\left(\arcsin\left(\frac{1}{n}\right)\right) = \sqrt{n^2-1}$

$\left[\cos(\arcsin x) = \sqrt{1-x^2}\right]$

$\Rightarrow$ B or D.

$n > 1$ always $\Rightarrow \sqrt{n^2+1} > 1 \Rightarrow \cancel{D}$



90. Two thin, concentric, spherical conducting shells are arranged as shown in the figure above. The inner shell has radius a, charge $+Q$, and is at zero electric potential. The outer shell has radius b and charge $-Q$. If r is the radial distance from the center of the spheres, what is the electric potential in region I $(a < r < b)$ and in region II $(r > b)$?

	Region I	Region II
(A)	$\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$	0
(B)	$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right)$	0
(C)	$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$	$-\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$
(D)	$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right)$	$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$
(E)	$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$	$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

the question is about V!

- $a < r < b$ V should depend on a.
- $b < r$ " , a and b

$\Rightarrow$ answer d).

using Gauss law we find:

- $r < a$ $\quad E_1 = 0$
- $a < r < b$ $\quad E_2 = \frac{Q}{4\pi\varepsilon_0 r^2}$ $\quad \vec{E} = -\vec{\nabla} V$
- $r > b$ $\quad E_3 = 0$ $\quad V = \int \vec{E} \cdot dr$

91. In static electromagnetism, let $\mathbf{E}$, $\mathbf{B}$, $\mathbf{J}$, and ρ be the electric field, magnetic field, current density, and charge density, respectively. Which of the following conditions allows the electric field to be written in the form $\mathbf{E} = -\nabla\phi$, where ϕ is the electrostatic potential?

$$\vec{\nabla} \times \vec{\nabla} . \vec{f} = \vec{0} .$$

(A) $\nabla \cdot \mathbf{J} = 0$
(B) $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$
(C) $\nabla \times \mathbf{E} = \mathbf{0}$
(D) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$
(E) $\nabla \cdot \mathbf{B} = 0$

---

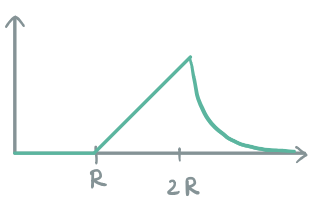
92. A long, straight, hollow cylindrical wire with an inner radius R and an outer radius $2R$ carries a uniform current density. Which of the following graphs best represents the magnitude of the magnetic field as a function of the distance from the center of the wire?

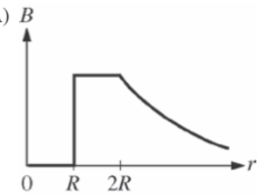
Ampère's law: $\oint \vec{B} . \vec{dl} = \mu_0 I_{enclosed}$

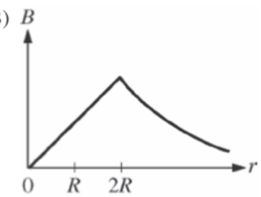
$r < R \Rightarrow 2\pi r B = 0 \Rightarrow B = 0$

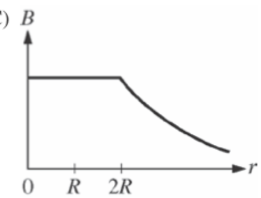
$R < r < 2R \Rightarrow 2\pi r B = \mu_0 J \times \pi r^2 \Rightarrow B \propto r$

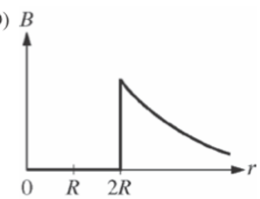
$2R < r \Rightarrow 2\pi r B = \mu_0 I \Rightarrow B \propto \frac{1}{r}$

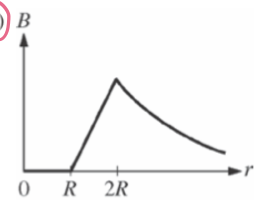


(A)


(B)


(C)


(D)


(E)


93. A parallel-plate capacitor has plate separation d. The space between the plates is empty. A battery supplying voltage V_0 is connected across the capacitor, resulting in electromagnetic energy U_0 stored in the capacitor. A dielectric, of dielectric constant κ, is inserted so that it just fills the space between the plates. If the battery is still connected, what are the electric field E and the energy U stored in the dielectric, in terms of V_0 and U_0?

	E	U
(A)	$\frac{V_0}{d}$	U_0
(B)	$\frac{V_0}{d}$	κU_0
(C)	$\frac{V_0}{d}$	$\kappa^2 U_0$
(D)	$\frac{V_0}{\kappa d}$	U_0
(E)	$\frac{V_0}{\kappa d}$	κU_0

$E = \frac{V_c}{d}$ $\qquad U = \frac{1}{2} C V_c^2$

since $V_c = V_0 \Rightarrow E = \frac{V_0}{d}$

$C_1 = \frac{\varepsilon_0 A}{d}$, $C_2 = \frac{\varepsilon_r A}{d} = k \frac{\varepsilon_0 A}{d} = k C_1$

hence $E_1 = E_2 = \frac{V_0}{d}$ and $U_2 = k U_1$

Interferences:

- add two waves with same λ, f, v (same medium)

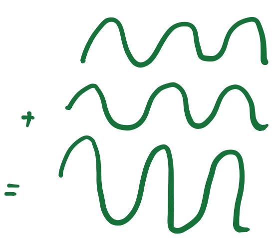
$$y_1(x_1,t) = A_1 \sin\left(\frac{2\pi t}{T} \pm \frac{2\pi x_1}{\lambda} + \phi_0^{(1)}\right)$$

$$y_2(x_2,t) = A_2 \sin\left(\frac{2\pi t}{T} \pm \frac{2\pi x_2}{\lambda} + \phi_0^{(2)}\right)$$

[now we take $A_1 = A_2 = A$ and right moving = away from source]

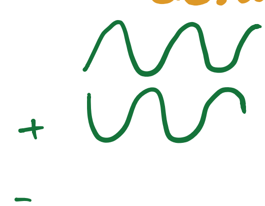
$$\boxed{\Delta\phi = \phi_1 - \phi_2 = -\frac{2\pi\Delta x}{\lambda} + \Delta\phi_0}$$

constructive:



$\Delta\phi = 2n\pi$

destructive:



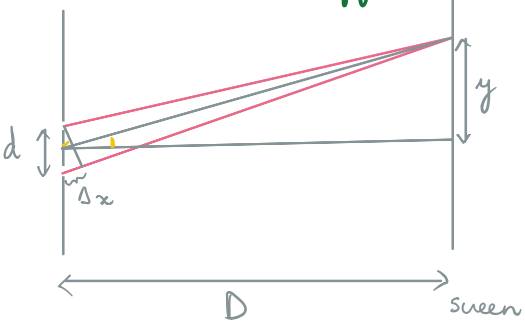
$\Delta\phi = (2n+1)\pi$

IF $\Delta\phi_0 = 0$

$\Delta x = n\lambda$

$\Delta x = \left(n+\frac{1}{2}\right)\lambda$

Double slit experiment:
(+ diffraction)



$$\tan\theta = \frac{y}{D} \qquad \tan\theta \approx \theta$$

$$\sin\theta = \frac{\Delta x}{d} \qquad \sin\theta \approx \theta$$

$$\frac{y}{D} = \frac{\Delta x}{d}$$

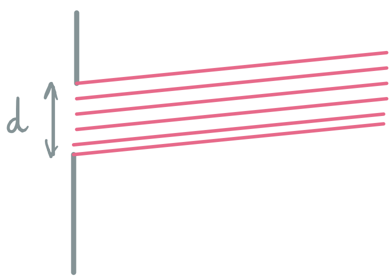
constructive interference : $\Delta x = n\lambda$

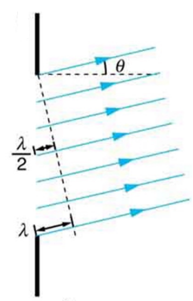
$$d = \frac{n\lambda}{y} D$$

destructive interference : $\Delta x = \left(n+\frac{1}{2}\right)\lambda$

$$d = \frac{\left(n+\frac{1}{2}\right)\lambda}{y} D$$

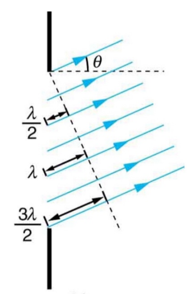
diffraction:





$\sin\theta = \frac{\lambda}{D}$

Dark



$\sin\theta = \frac{3\lambda}{2D}$

Bright

destructive interferences:

$\Delta x = n\lambda$

constructive interference

$\Delta x = \left(n+\frac{1}{2}\right)\lambda$

$$\boxed{\sin\theta = \frac{\Delta x}{d}}$$

Optics:

		f	i
mirror	convex	$-$	only virtual
	concave	$+$	i real $(+)$ if $o > f$ i virtual $(-)$ if $o < f$
lense	convex	$+$	i real $(+)$ if $o > f$ i virtual $(-)$ if $o < f$
	concave	$-$	only virtual

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \qquad M = \frac{h_i}{h_o} = -\frac{i}{o}$$

maxwell equations : (static)

(1) $\vec{\nabla}.\vec{B} = 0$ (3) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$ (4) $\vec{\nabla}\times\vec{E} = \vec{0}$

---

(1) $\vec{\nabla}.\vec{B} = 0 \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$ because $\vec{\nabla}.\vec{\nabla}\times\vec{F} = 0$

(2) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Rightarrow \iint \vec{\nabla}\times\vec{B}.d\vec{A} = \iint \mu_0 \vec{J} \underset{\text{Stockes}}{\Rightarrow} \oint \vec{B}.d\vec{l} = \mu_0 I_{enc}$

(3) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \iiint \vec{\nabla}.\vec{E}.dV = \iiint \frac{\rho}{\varepsilon_0} \underset{\text{Gauss}}{\Rightarrow} \iint \vec{E}.d\vec{S} = \frac{Q_{ins}}{\varepsilon_0}$

(4) $\vec{\nabla}\times\vec{E} = 0 \Rightarrow \vec{E} = -\vec{\nabla}.\phi$ because $\vec{\nabla}\times\vec{\nabla}f = 0$

maxwell equations in general:

(1) $\vec{\nabla}.\vec{B} = 0$ (3) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$ (4) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

wave equation : (4) and (2) $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\left(\frac{\partial \vec{B}}{\partial t}\right)$
(no sources)

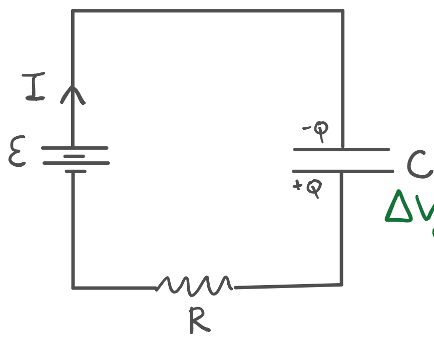
$\Rightarrow \vec{\nabla}.\underbrace{\cancel{(\vec{\nabla}.\vec{E})}}_{0} - (\vec{\nabla}.\vec{\nabla})\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$

$-\nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$

$\Rightarrow \boxed{\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}}$

RC circuit :

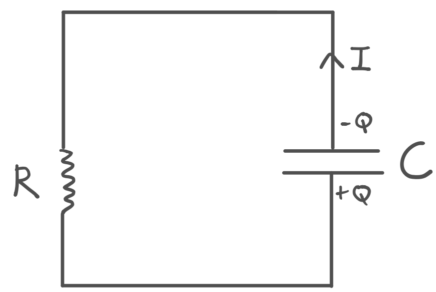
charge :



$$\Delta V_c = \frac{Q(t)}{C}$$

$$\mathcal{E} + \frac{Q(t)}{C} - RI = 0 \qquad I = \frac{dQ}{dt}$$

discharge :

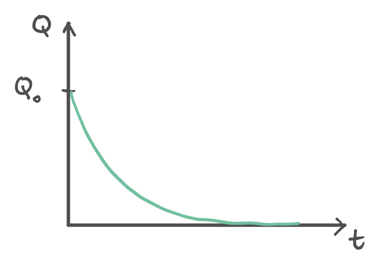


$$\Delta V_R = -\Delta V_c$$

$$RI = -\frac{Q}{C}$$

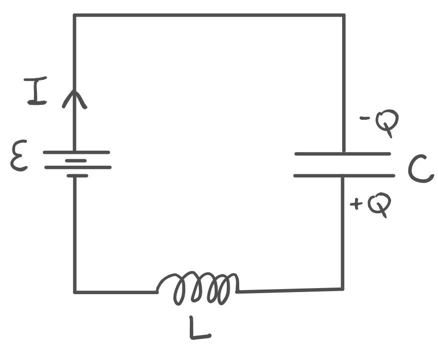
$$R\frac{dQ}{dt} = \frac{-Q}{C} \Rightarrow \frac{dQ}{dt} = \frac{-Q}{RC}$$

$$\boxed{Q(t) = Q_0 e^{-\frac{t}{RC}}}$$

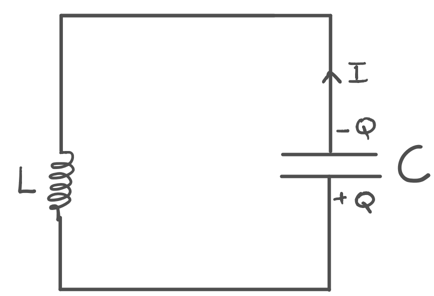


LC circuit

charge :



discharge :



$$\Delta V_c = \frac{Q}{C} \qquad V_L = L\frac{dI}{dt}$$