Physics GRE Strategies and Tips

David Wittman
University of California, Davis
(borrowing heavily from Damien Martin)

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Strategies

- **triage:** too many questions to analyze each in detail, so
  - skip anything that looks hard, or
  - make a first pass all the way thru marking your priorities
  - remember: you* only need to nail about 40 questions, so choose wisely!
- **eliminate options** based on dimensional analysis or functional forms (exponentials, sines, cosines)
- **use expansions**, esp. $(1+x)^n \approx 1 + nx$ for small x
- use **limiting cases/special cases** to see if an answer makes sense.
- powers of ten estimation sometimes helps
- know scales of things, at least in your area of expertise (eg wavelength/freq of visible light, mass ratios of common particles if you like particles, masses of stars and galaxies if you like astro, etc)

*see gradcafe.com to fine-tune this estimate

Triage Example

During a hurricane, a 1,200 Hz warning siren on the town hall sounds. The wind is blowingat 55 m/s in a direction from the siren toward a person 1 km away. With what frequency does the sound wave reach the person?

A. 1,000 Hz
B. 1,030 Hz
C. 1,200 Hz
D. 1,400 Hz
E. 1,440 Hz

Which of the following ions CANNOT be used as a dopant in germanium to make an n-type semiconductor?

(a) As
(b) P
(c) Sb
(d) B
(e) N

Study Strategies

- know your first-year general physics really well!
- know your "modern physics" general physics really well
- if you can "overlearn" everything in, eg, Halliday and Resnick, you can nail most questions...*and you don't need to nail the rest*.
- other texts worth going through: Griffiths E&M, Griffiths quantum (concentrate on harmonic osc, infinite square well, spin systems, expectation values), Schroeder's Thermal Physics
- recommended book: *Conquering the Physics GRE*
- GREphysics.net also a good resource

Scope of Study

Bare bones thermal: Know the definition of partition function, probability of a state, how to find an expectation value.

$$
\begin{aligned}
Z &= \sum_{\text{state i}} e^{-\beta E_i} = \sum_{\text{energies j}} g_j e^{-\beta E_j} \\
\text{Prob(State i)} &= \frac{1}{Z} e^{-\beta E_i} \\
\langle X \rangle &= \sum_{\text{state i}} X_i P(X_i) = \frac{1}{Z} \sum_{\text{state i}} X_i e^{-\beta E_i}
\end{aligned}
$$

where $\beta \equiv ???$

Scope of Study

Bare bones quantum: know the "modern physics" course: how to find probability of a state, know how to find expectation values, know the special systems, know spin-addition rules.

Special systems: particle in box, harmonic oscillator, two spin-$1/2$ particles.

Scope of Study

Bare bones analytical mechanics:

know *how to find* the Hamiltonian, Hamilton's equations, Lagrangian, and the Euler Lagrange equations for:

- a particle in a gravitational field
- a charged particle in a uniform electric field
- a pendulum.

Don't just memorize the results: if you can **do** these three systems you will be aware of the pattern.

Remember: GRE questions are typically short—they won't ask questions that require crazy-detailed calculations!

Four Example Problems

$$C = 3kN_A\left(\frac{h\nu}{kT}\right)^2 \frac{e^{h\nu/kT}}{(e^{h\nu/kT}-1)^2}$$

65. Einstein's formula for the molar heat capacity C of solids is given above. At high temperatures, C approaches which of the following?

(A) 0

(B) $3kN_A\left(\frac{h\nu}{kT}\right)$

(C) $3kN_A h\nu$

(D) $3kN_A$

(E) $N_A h\nu$

Dimensions here → $3kN_A$; This quantity is dimensionless ← $e^{h\nu/kT}$

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Call $x = \frac{h\nu}{kT}$

$$\begin{aligned} C &= 3kN_A x^2 \frac{e^x}{(e^x-1)^2} \\ &= 3kN_a x^2 \left[\frac{1+x+\dots}{((1+x+\dots-1)^2}\right] \\ &= 3kN_a x^2 \left[\frac{1}{x^2}+\dots\right] \\ &= 3kN_a + \dots \end{aligned}$$

A distant galaxy is observed to have its H-beta line shifted to a wavelength of 480nm from its laboratory value of 434nm. Which is the best approximation to the velocity of the galaxy? (Note: 480/434 ~ 1.1)

a) 0.01c
b) 0.05c
c) 0.1c
d) 0.32c
e) 0.5c

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$$\lambda_{\rm obs} = \lambda_{\rm emit}\sqrt{\frac{c+v}{c-v}}$$

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} = \sqrt{\frac{1+(v/c)}{1-(v/c)}} \approx \sqrt{(1+(v/c))^2}$$

$$v \approx \left(\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} - 1\right) c$$

14. An 8-centimeter-diameter by 8-centimeter-long NaI(Tl) detector detects gamma rays of a specific energy from a point source of radioactivity. When the source is placed just next to the detector at the center of the circular face, 50 percent of all emitted gamma rays at that energy are detected. If the detector is moved to 1 meter away, the fraction of detected gamma rays drops to

(A) 10^{-4}
(B) 2×10^{-4}
(C) 4×10^{-4}
(D) $8\pi \times 10^{-4}$
(E) $16\pi \times 10^{-4}$

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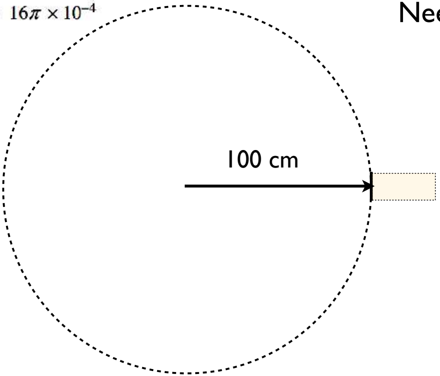

Need fraction of area covered by sensor

$$A_{\text{sensor}} = \pi(4\ \text{cm})^2 = 16\pi\ \text{cm}^2$$
$$A_{\text{sphere}} = 4\pi(100\ \text{cm}^2) = 4\pi \times 10^4\ \text{cm}^2$$

$$\frac{A_{\text{sensor}}}{A_{\text{sphere}}} = \frac{16\pi}{4\pi \times 10^4} = 4 \times 10^{-4}$$

2. The coefficient of static friction between a small coin and the surface of a turntable is 0.30. The turntable rotates at 33.3 revolutions per minute. What is the maximum distance from the center of the turntable at which the coin will not slide?

(A) 0.024 m
(B) 0.048 m
(C) 0.121 m
(D) 0.242 m
(E) 0.484 m

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max friction = "centrifugal" force [http://xkcd.com/123/]

$\mu N = \mu m g = m\omega^2 r$ so $r = \dfrac{\mu g}{\omega^2}$

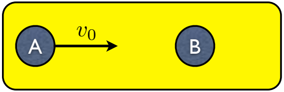
$$\omega = \frac{2\pi}{T} \text{ (don't use!)}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \times 33.3}{60 \text{ s}} \sim \pi \text{ s}^{-1} \quad \Rightarrow \omega^2 \sim 10 \text{ s}^{-2}$$ therefore

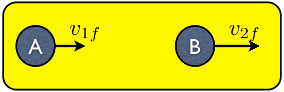
$$r \sim \mu \text{ m} \sim 0.3 \text{ m}$$

Things to know (they always seem to come up)

1) Elastic collision formula



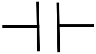
Before collision



After collision

$$
\begin{aligned}
v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_0 \\
v_{2f} &= \frac{2m_1}{m_1 + m_2} v_0
\end{aligned}
$$

2) The limiting behavior of capacitors and inductors in DC

 acts like (while uncharged)

 (while fully charged)

(e.g. high pass filter question)

3) Virial theorem (and the quick way to get it)

$$F(r) = Ar^{+n} \Rightarrow V = \frac{A}{1+n} r^{1+n} = \frac{F(r)}{1+n} r$$

$$\frac{mv^2}{r} = F(r) \Rightarrow \frac{1}{2} mv^2 = \frac{1}{2} F(r) r$$

$$\langle KE \rangle = \frac{1+n}{2} \langle V \rangle$$

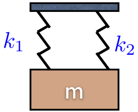
4) The Bohr formula (or know how to get it quickly)

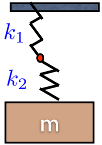
$$E = -\frac{Z^2 (ke^2)^2 m}{2\hbar^2 n^2}$$

m is reduced mass!

(To get levels for e.g. positronium, same formula but use reduced mass for that system)

5) Combining masses, springs, capacitors, resistors





Can you find k_{equiv} ?

Frequency of oscillation?

Know reduced mass!

6) Making problems look like a harmonic oscillator

$$\omega^2 = \frac{(d^2V/dx^2)|_{\min}}{m}$$

7) Remember spectroscopic notation (ugh)

$$^{2s+1}(\text{orbital angular momentum symbol})_j$$

and the selection rules for an electric dipole

8) Know the *pattern* of spherical harmonics

Y_ℓ^m

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}} \quad (\ell = 0)$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{-i\varphi}$$

$$Y_1^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta \quad (\ell = 1)$$

$$Y_1^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\,\cos\theta\, e^{-i\varphi}$$

$$Y_2^0(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1) \quad (\ell = 2)$$

$$Y_2^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\,\cos\theta\, e^{i\varphi}$$

$$Y_2^2(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\varphi}$$

Too detailed ...... !

(But if you can remember these, congratulations)

8) Know the *pattern* of spherical harmonics

$$Y_\ell^m$$

m – magnetic quantum number $(-\ell, -\ell+1, \ldots, \ell)$

ℓ – orbital quantum number $(0, 1, 2, \ldots)$

Y_ℓ^m contains φ dependence of the form $e^{im\phi}$

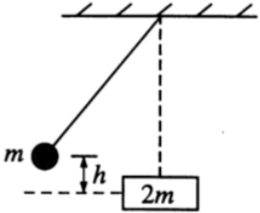
Y_ℓ^m contains ℓ dependence of the form $\sin^\ell\theta, \quad \sin^{\ell-1}\theta\cos\theta, \ldots$

(i.e. can write as ℓ sines or cosines mulitpled, or as $\sin(\ell\theta)$, $\cos(\ell\theta)$.)

Compare these rules to the spherical harmonics listed one slide ago.

Selected items from these eight in more detail

Elastic collisions may be obfuscated



7. As shown above, a ball of mass m, suspended on the end of a wire, is released from height h and collides elastically, when it is at its lowest point, with a block of mass $2m$ at rest on a frictionless surface. After the collision, the ball rises to a final height equal to

(A) 1/9 h
(B) 1/8 h
(C) 1/3 h
(D) 1/2 h
(E) 2/3 h

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Quick Bohr (semi-classical) derivation

Electron traveling in a circle:

$$\frac{mv^2}{r} = \frac{k(Ze)(e)}{r^2}$$

Angular momentum is quantized:

$$L = pr = mvr = n\hbar$$

Put together to find r (Bohr radius!)

$$\frac{1}{r} = \frac{kZe^2}{mv^2r^2} = \frac{kZme^2}{(mvr)^2} = \frac{kZme^2}{n^2\hbar^2}$$

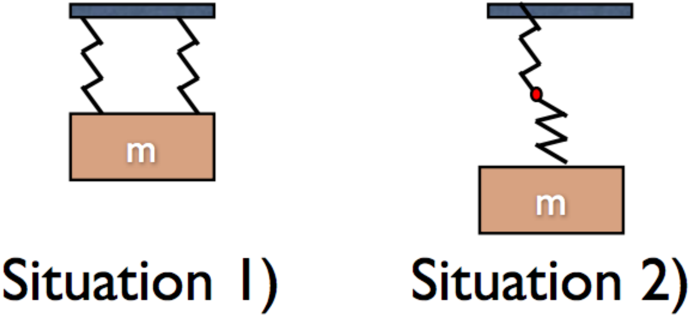
Potential energy:

$$PE = -\frac{kZe^2}{r} = -\frac{k^2Z^2me^4}{n^2\hbar^2}$$

Virial thm: $< E >= - < PE > /2$

Combining things in series or parallel

The effective [R,C,k,...] is always found by either adding the [R,C,k,...] or by adding their *inverses*. Why?



If each individual spring has spring constant k, what is k_{eff} in each situation?

Now apply that in context

Situation 1) Situation 2)

Two different ways of connecting a mass *m* to two *identical springs* with spring constant k are shown above. If we denote the frequency of oscillation in situation 1 by f_1 and the frequency of oscillation in situation 2 by f_2 then f_1/f_2 is:

a) 4
b) 2
c) 1/2
d) 1/4
e) depends on m and / or k

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b) 2
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(Hint: can pretend k_1 and k_2 are not the same to take limits to determine formula for k_{eff})

$$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{k_{\text{eff},1}/m}{k_{\text{eff},2}/m}} = \sqrt{\frac{k_{\text{eff},1}}{k_{\text{eff},2}}} = \sqrt{\frac{2k}{k/2}} = 2$$

A perhaps surprising application of this principle

66. A sample of radioactive nuclei of a certain element can decay only by γ-emission and β-emission. If the half-life for γ-emission is 24 minutes and that for β-emission is 36 minutes, the half-life for the sample is

(A) 30 minutes
(B) 24 minutes
(C) 20.8 minutes
(D) 14.4 minutes
(E) 6 minutes

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$$t_{1/2} \stackrel{?}{=} t_\gamma + t_\beta \qquad \text{or} \qquad \frac{1}{t_{1/2}} \stackrel{?}{=} \frac{1}{t_\gamma} + \frac{1}{t_\beta}$$

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$$\begin{aligned} \frac{1}{t_{1/2}} = \frac{1}{t_\gamma} + \frac{1}{t_\beta} &= \frac{1}{24} + \frac{1}{36} \\ &= \frac{1}{6}(\frac{1}{4} + \frac{1}{6}) \\ &= \frac{1}{6}\frac{10}{24} \\ \Rightarrow t_{1/2} = \frac{24 \times 6}{10} &= \frac{144}{10} = 14.4 \text{ min} \end{aligned}$$

Another perhaps surprising application

“Reduced” mass:

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

Application of reduced mass

What is the emission energy from a photon going from n = 3 to n = 1 in *positronium* (one electron and one positron orbiting one another)?

Rule for Hydrogen like atoms: $E_n = \frac{-13.6 \text{ eV}}{n^2} Z^2$

But 13.6 is proportional to the *reduced* mass $m = m_{\text{electron}}$ in Hydrogen

In positronium $m = m_e/2$, so we have to halve the 13.6

$$\boxed{E_n = -\frac{6.8 \text{ eV}}{n^2}}, \qquad \text{(positronium energy levels)}$$

$$E_{\text{photon}} = E_3 - E_1 = 6.8 \text{ eV} \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = 6.8 \text{ eV} \times \frac{8}{9} \approx 6 \text{ eV}$$

Making problems look like a harmonic oscillator

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

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Let y be the distance from the minimum. Expanding about the minimum we have:

$$V(y) = V_{\text{min}} + 0y + \frac{1}{2}\frac{d^2V}{dy^2}\Big|_{\text{min}} y^2 + \ldots$$

0 (because min)

Just a number, not a function

Force is

$$F = -\frac{dV}{dy} = -\frac{d^2V}{dy^2}\Big|_{\text{min}} y + \ldots$$

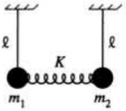
SHM with "spring constant" $k = d^2V/dy^2$ evaluated at min!

spring constant $= -dk^2\sin(kx) = +dk^2$ evaluated at min

$$f = 2\pi\sqrt{\frac{\text{spring const.}}{m}} = 2\pi\sqrt{\frac{dk^2}{m}}$$

See *Conquering the Physics GRE* for many more tips! (eg, a much more detailed list of things that are useful to memorize)

Extra Slides



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical lengths ℓ, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of this system?

(A) $\sqrt{g/\ell}$

(B) $\sqrt{\frac{K}{m_1 + m_2}}$

(C) $\sqrt{\frac{K}{m_1} + \frac{K}{m_2}}$

(D) $\sqrt{\frac{g}{\ell} + \frac{K}{m_1} + \frac{K}{m_2}}$

(E) $\sqrt{\frac{2g}{\ell} + \frac{K}{m_1 + m_2}}$

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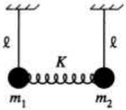

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$$f_{\text{high}} \xrightarrow{K \text{ large}} ??$$

$$f_{\text{high}} \xrightarrow{K \text{ small}} ??$$

Reduced mass