The Northern California Physics GRE Bootcamp

Held at UC Davis, August 13-14, 2016

Damien Martin

Big picture (your exam)

TOTAL SCORE

Raw Score	Scaled Score	%
85-100	990	98
84	980	97
82-83	970	97
81	960	96
80	950	95
78-79	940	95
77	930	94
75-76	920	92
74	910	91
73	900	90
71-72	890	89
70	880	88
68-69	870	87
67	860	86
65-66	850	84
64	840	83
63	830	82
61-62	820	81
60	810	79
58-59	800	78
57	790	76
55-56	780	74
54	770	72
53	760	71
51-52	750	69
50	740	67
48-49	730	65
47	720	63
45-46	710	61
44	700	59
43	690	57
41-42	680	54
40	670	53
38-39	660	50
37	650	48
35-36	640	45
34	630	44
33	620	41
31-32	610	39
30	600	37
28-29	590	34
27	580	32
26	570	29
24-25	560	27
23	550	25
21-22	540	22
20	530	20
18-19	520	18
17	510	16
16	500	13
14-15	490	11
13	480	10
11-12	470	7
10	460	6
8-9	450	5
7	440	4
6	430	3
4-5	420	1
3	410	1
1-2	400	1
0	390	1

*The percent scoring below the scaled score is based on the performance of 10,947 examinees who took the Physics Test between July 1, 2000, and June 30, 2003.

QUESTION Number	Answer	P +	TOTAL C	I	QUESTION Number	Answer	P +	TOTAL C	I
1	C	54			51	B	45		
2	D	30			52	C	12		
3	D	71			53	B	32		
4	C	62			54	C	77		
5	D	28			55	E	62		
6	E	34			56	D	54		
7	B	89			57	A	68		
8	D	65			58	B	58		
9	A	63			59	B	87		
10	A	53			60	D	55		
11	A	28			61	C	18		
12	E	40			62	A	35		
13	B	42			63	D	52		
14	C	27			64	A	56		
15	A	68			65	D	44		
16	D	14			66	D	33		
17	B	81			67	E	19		
18	A	45			68	E	51		
19	B	36			69	B	26		
20	E	49			70	B	53		
21	B	60			71	D	32		
22	A	54			72	E	39		
23	C	45			73	D	43		
24	C	86			74	D	50		
25	E	48			75	E	57		
26	C	30			76	C	49		
27	A	82			77	E	44		
28	E	61			78	E	52		
29	C	63			79	D	69		
30	A	44			80	D	28		
31	A	53			81	B	50		
32	D	62			82	D	16		
33	D	31			83	C	30		
34	C	23			84	D	26		
35	E	82			85	A	25		
36	E	70			86	E	24		
37	D	36			87	A	42		
38	D	35			88	C	42		
39	D	45			89	E	37		
40	D	40			90	A	33		
41	E	66			91	B	41		
42	C	64			92	E	45		
43	D	39			93	C	42		
44	D	54			94	E	29		
45	B	50			95	A	42		
46	E	29			96	E	13		
47	B	46			97	E	20		
48	C	57			98	A	72		
49	E	61			99	D	20		
50	B	50			100	B	72		

(24) 20-35 percentile

(6) <20 percentile

Big picture (1993-1996)

TOTAL SCORE

Raw Score	Scaled Score	%	Raw Score	Scaled Score	%
67-99	990	97	30-31	690	59
65-66	980	96	29	680	57
64	970	96	28	670	55
63	960	95	27	660	53
62	950	95	26	650	50
61	940	94	24-25	640	48
59-60	930	93	23	630	45
58	920	92	22	620	43
57	910	91	21	610	40
56	900	91	19-20	600	38
54-55	890	90	18	590	35
53	880	89	17	580	32
52	870	88	16	570	29
51	860	86	15	560	27
50	850	85	13-14	550	25
48-49	840	84	12	540	23
47	830	83	11	530	20
46	820	82	10	520	18
45	810	81	9	510	15
44	800	79	7-8	500	13
42-43	790	77	6	490	11
41	780	76	5	480	9
40	770	74	4	470	7
39	760	73	3	460	6
38	750	71	1-2	450	4
36-37	740	69	0	440	3
35	730	67			
34	720	65			
33	710	63			
32	700	61			

*Percentage scoring below the scaled score is based on the performance of 11,322 examinees who took the Physics Test between October 1, 1993, and September 30, 1996.

QUESTION Number	QUESTION Answer	P +	TOTAL C	TOTAL I
1	B	73		
2	B	29		
3	B	55		
4	A	34		
5	B	29		
6	B	43		
7	A	22		
8	A	37		
9	A	40		
10	B	47		
11	D	36		
12	C	36		
13	B	37		
14	D	66		
15	E	12		
16	B	20		
17	E	40		
18	C	77		
19	B	17		
20	D	20		
21	C	27		
22	C	26		
23	D	24		
24	D	70		
25	E	38		
26	C	13		
27	D	49		
28	D	40		
29	E	58		
30	C	28		
31	E	65		
32	B	41		
33	C	56		
34	D	31		
35	A	79		
36	D	46		
37	D	53		
38	D	39		
39	E	54		
40	A	21		
41	A	28		
42	B	52		
43	C	17		
44	A	48		
45	E	45		
46	C	36		
47	C	28		
48	C	30		
49	C	22		
50	C	33		

QUESTION Number	QUESTION Answer	P +	TOTAL C	TOTAL I
51	B	70		
52	C	15		
53	C	34		
54	B	18		
55	A	30		
56	C	42		
57	C	41		
58	E	22		
59	B	36		
60	B	9		
61	B	26		
62	C	13		
63	A	56		
64	C	26		
65	D	44		
66	E	25		
67	C	28		
68	E	61		
69	A	14		
70	D	14		
71	A	20		
72	A	29		
73	C	34		
74	B	21		
75	D	27		
76	B	52		
77	D	12		
78	E	36		
79	D	22		
80	C	31		
81	B	27		
82	E	15		
83	D	15		
84	D	20		
85	B	15		
86	B	36		
87	A	6		
88	B	57		
89	D	18		
90	**	**		
91	E	25		
92	D	15		
93	D	26		
94	D	28		
95	E	23		
96	A	28		
97	E	11		
98	D	39		
99	B	44		
100	C	51		

(35) 20-35 percentile

(18) <20 percentile

Big tips and tricks

* Multiple passes through the exam

* Dimensional analysis (which answers make sense?)
Other hint -- look at exponentials, sines, cosines, ...

* Expansions, in particular $(1+x)^n = 1 + n x + ...$

* Limiting cases (e.g. make parameters go to 0 or infinity)

* Special cases (e.g. looking at circles)

* Powers of ten estimation

* Know scales of things [wavelength / freq of visible light, binding energies of nuclei, mass ratios of common particles (up to muon, pion), ..., mass of stars, mass of galaxies, ...]

Okay to specialize on scales

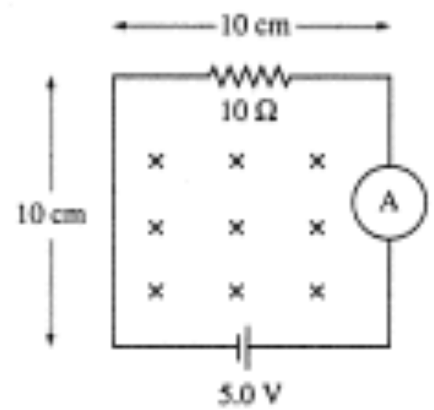
Big tips and tricks -- material

* Know your "first year" general physics really well
  - Newtonian mechanics in particular

* Know your "modern physics" general physics really well
  (usually Sophmore class)

* Worth going through
  Griffiths: Intro to electromagnetism
  Griffiths: Intro to quantum mechanics
  (Concentrate on harmonic osc, infinite square well, spin systems, expectation values)
  Schroeder: Thermal physics

* Look at the archive of monthly problems in *The Physics Teacher* (if you have access to a university library)



2. The circuit shown above is in a uniform magnetic field that is into the page and is decreasing in magnitude at the rate of 150 tesla/second. The ammeter reads

(A) 0.15 A
(B) 0.35 A
(C) 0.50 A
(D) 0.65 A
(E) 0.80 A

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GO ON TO THE NEXT PAGE.

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Evaluate whether question is "special" or "first year"

67. A black hole is an object whose gravitational field is so strong that even light cannot escape. To what approximate radius would Earth (mass $= 5.98 \times 10^{24}$ kilograms) have to be compressed in order to become a black hole?

(A) 1 nm
(B) 1 μm
(C) 1 cm
(D) 100 m
(E) 10 km

28

69

Specific tips and tricks -- material

Bare bones thermal:
Know the definition of partition function, probability of a state, how to find an expectation value

$$Z = \sum_{\text{state i}} \exp(-\beta E_i) = \sum_{\text{energies j}} g_j \exp(-\beta E_j)$$

$$\text{Prob(State i)} = \frac{1}{Z} e^{-\beta E_i}$$

$$\langle X \rangle = \sum_{\text{state i}} X_i P(X_i) = \frac{1}{Z} \sum_{\text{state i}} X_i \exp(-\beta E_i)$$

Bare bones quantum:
Know the "modern physics" course, know how to find probability of a state, know how to find expectation values, know the special systems, know spin-addition rules

special systems: *particle in box, harmonic oscillator, two spin 1/2 particles*

Specific tips and tricks -- material

Bare bones (advanced) classical:
Know ***how to find*** *the Hamiltonian, Hamilton's equations, Lagrangian, and the Euler Lagrange equations for a particle in a gravitational field, charged particle in a uniform electric field, and a pendulum.*

Don't just memorize the results, if you can do these three systems you will be aware of the pattern.

Remember: GRE questions are typically short -- cannot get you to do any crazy calculations!

$$C = 3kN_A\left(\frac{h\nu}{kT}\right)^2 \frac{e^{h\nu/kT}}{(e^{h\nu/kT}-1)^2}$$

65. Einstein's formula for the molar heat capacity C of solids is given above. At high temperatures, C approaches which of the following?

(A) 0

(B) $3kN_A\left(\frac{h\nu}{kT}\right)$

(C) $3kN_A h\nu$

(D) $3kN_A$

(E) $N_A h\nu$

Dimensions here → $C = 3kN_A\left(\frac{hv}{kT}\right)^2 \frac{e^{hv/kT}}{(e^{hv/kT}-1)^2}$ ← This quantity is dimensionless

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(A) 0

(B) $3kN_A\left(\frac{hv}{kT}\right)$

~~(C) $3kN_A hv$~~

(D) $3kN_A$

~~(E) $N_A hv$~~

Call $x = \frac{hv}{kT}$

$$
\begin{aligned}
C &= 3kN_A x^2 \frac{e^x}{(e^x-1)^2} \\
&= 3kN_a x^2 \left[\frac{1+x+\ldots}{((1+x+\ldots-1)^2}\right] \\
&= 3kN_a x^2 \left[\frac{1}{x^2}+\ldots\right] \\
&= 3kN_a + \ldots
\end{aligned}
$$

A distant galaxy is observed to have its H-beta line shifted to a wavelength of 480nm from its laboratory value of 434nm. Which is the best approximation to the velocity of the galaxy? (Note: 480/434 ~ 1.1)

a) 0.01c
b) 0.05c
c) 0.1c
d) 0.32c
e) 0.5c

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b) 0.05c
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$$\lambda_{\mathrm{obs}} = \lambda_{\mathrm{emit}} \sqrt{\frac{c+v}{c-v}}$$

$$\frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{emit}}} = \sqrt{\frac{1+(v/c)}{1-(v/c)}} \approx \sqrt{(1+(v/c))^2}$$

$$v \approx \left(\frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{emit}}} - 1 \right) c$$

14. An 8-centimeter-diameter by 8-centimeter-long NaI(Tl) detector detects gamma rays of a specific energy from a point source of radioactivity. When the source is placed just next to the detector at the center of the circular face, 50 percent of all emitted gamma rays at that energy are detected. If the detector is moved to 1 meter away, the fraction of detected gamma rays drops to

(A) 10^{-4}

(B) 2×10^{-4}

(C) 4×10^{-4}

(D) $8\pi \times 10^{-4}$

(E) $16\pi \times 10^{-4}$

27

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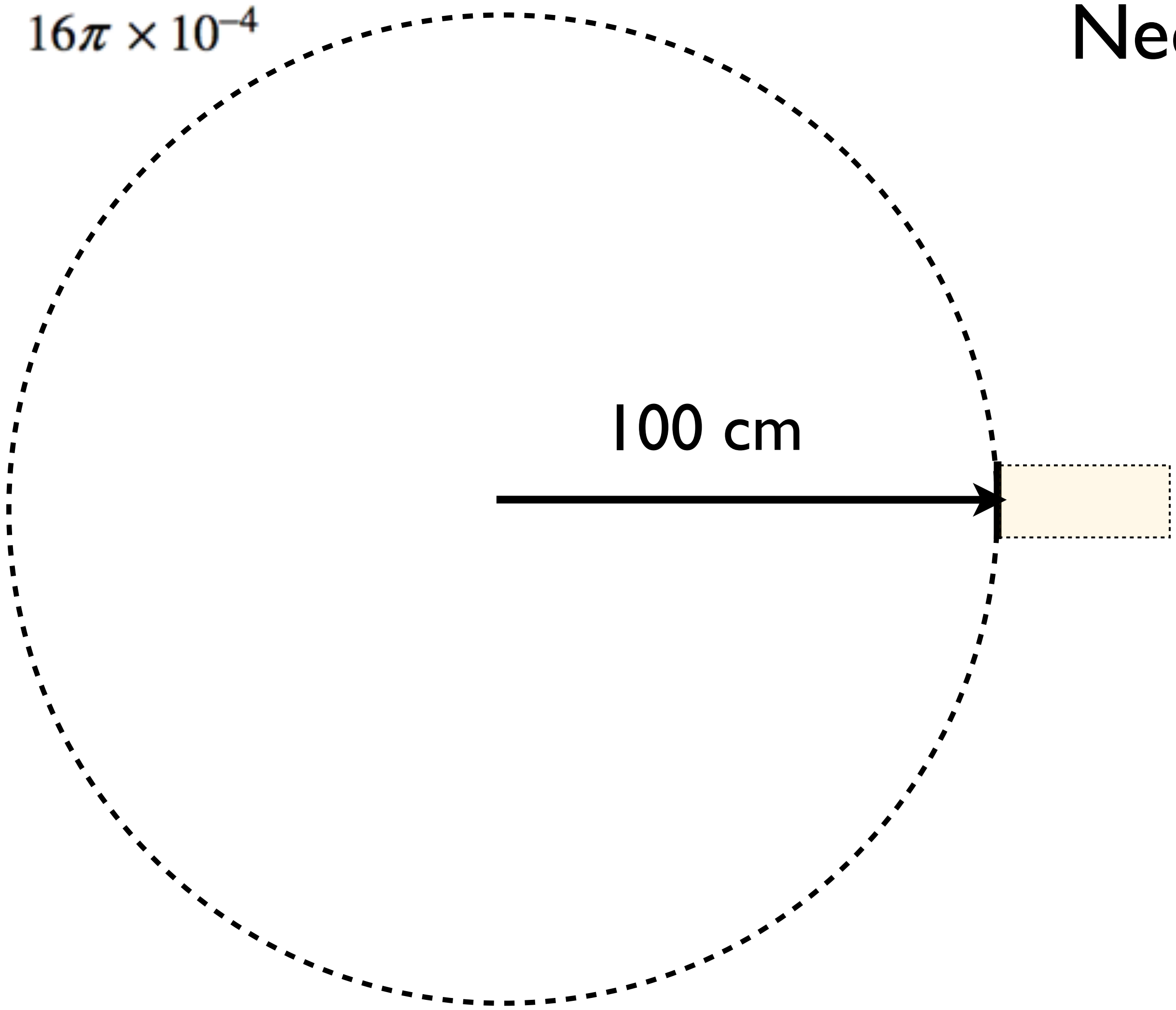

Need fraction of area covered by sensor

$$A_{\text{sensor}} = \pi(4\ \text{cm})^2 = 16\pi\ \text{cm}^2$$
$$A_{\text{sphere}} = 4\pi(100\ \text{cm}^2) = 4\pi \times 10^4\ \text{cm}^2$$

$$\frac{A_{\text{sensor}}}{A_{\text{sphere}}} = \frac{16\pi}{4\pi \times 10^4} = 4 \times 10^{-4}$$

2. The coefficient of static friction between a small coin and the surface of a turntable is 0.30. The turntable rotates at 33.3 revolutions per minute. What is the maximum distance from the center of the turntable at which the coin will not slide?

(A) 0.024 m
(B) 0.048 m
(C) 0.121 m
(D) 0.242 m
(E) 0.484 m

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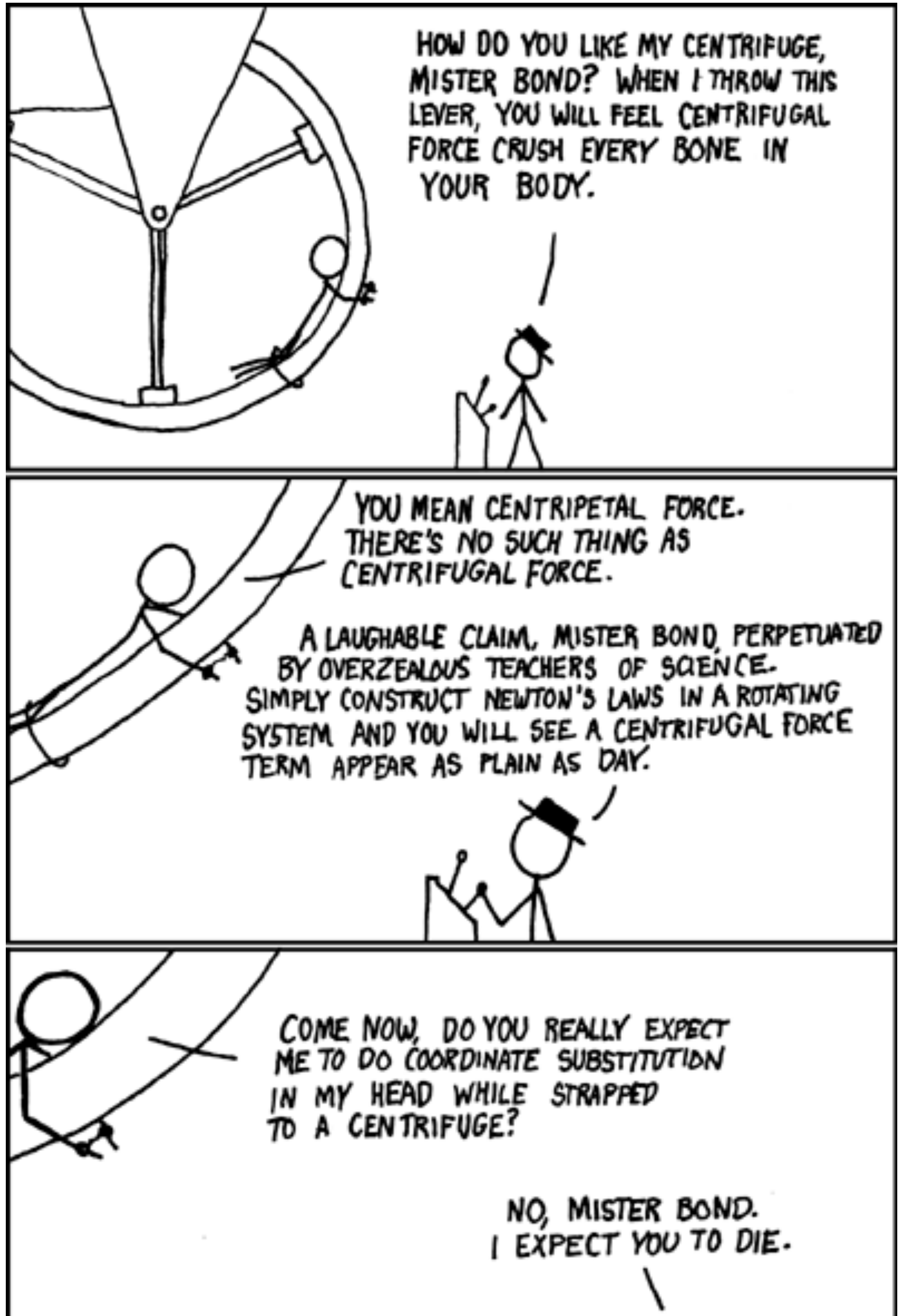
max friction = "centrifugal" force [http://xkcd.com/123/]

$\mu N = \mu m g = m\omega^2 r$ **so** $r = \frac{\mu g}{\omega^2}$

$$\omega = \frac{2\pi}{T} \text{ (don't use!)}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \times 33.3}{60 \text{ s}} \sim \pi \text{ s}^{-1} \qquad \Rightarrow \omega^2 \sim 10 \text{ s}^{-2}$$ therefore

$$r \sim \mu \text{ m} \sim 0.3 \text{ m}$$


HOW DO YOU LIKE MY CENTRIFUGE, MISTER BOND? WHEN I THROW THIS LEVER, YOU WILL FEEL CENTRIFUGAL FORCE CRUSH EVERY BONE IN YOUR BODY.
YOU MEAN CENTRIPETAL FORCE. THERE'S NO SUCH THING AS CENTRIFUGAL FORCE.
A LAUGHABLE CLAIM, MISTER BOND, PERPETUATED BY OVERZEALOUS TEACHERS OF SCIENCE. SIMPLY CONSTRUCT NEWTON'S LAWS IN A ROTATING SYSTEM AND YOU WILL SEE A CENTRIFUGAL FORCE TERM APPEAR AS PLAIN AS DAY.
COME NOW, DO YOU REALLY EXPECT ME TO DO COORDINATE SUBSTITUTION IN MY HEAD WHILE STRAPPED TO A CENTRIFUGE?
NO, MISTER BOND. I EXPECT YOU TO DIE.

Things to know (they always seem to come up)

1) Elastic collision formula

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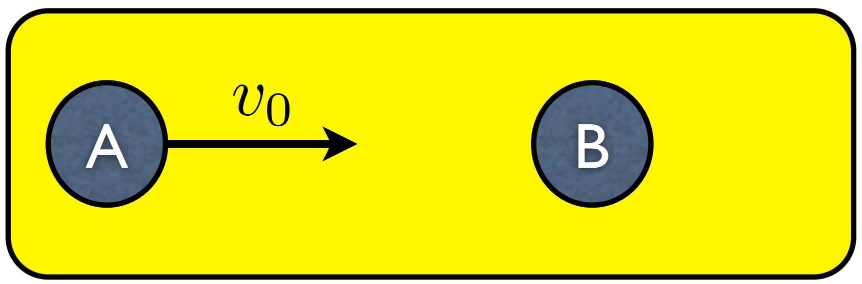
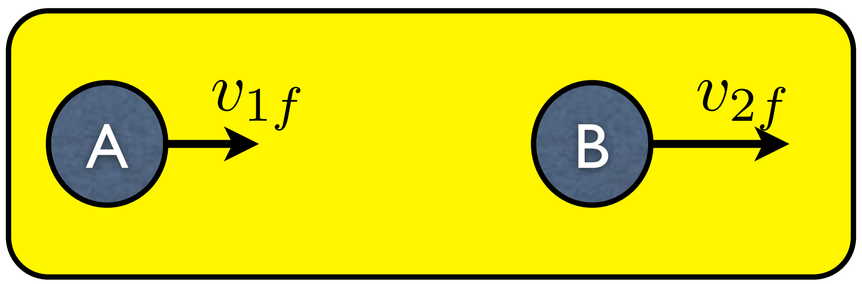


Before collision



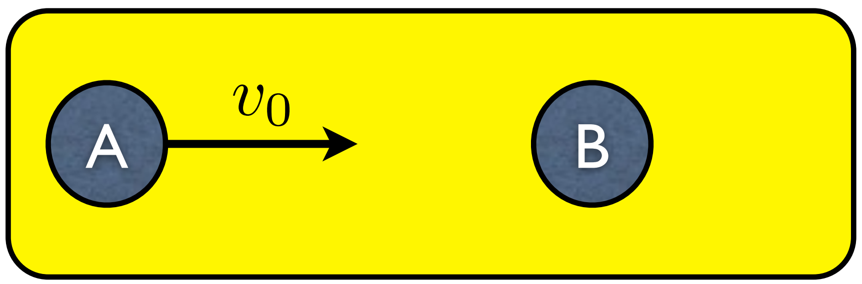
After collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_0$$

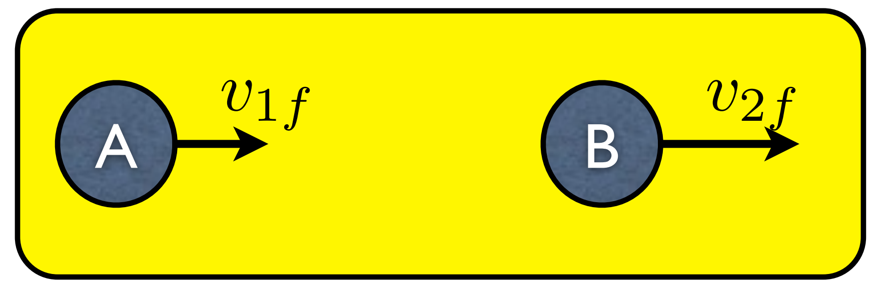
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_0$$

Things to know (they always seem to come up)

1) Elastic collision formula



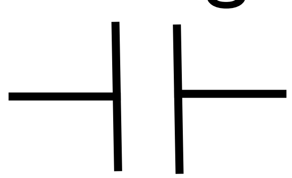
Before collision



After collision

$$
\begin{aligned}
v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_0 \\
v_{2f} &= \frac{2m_1}{m_1 + m_2} v_0
\end{aligned}
$$

2) The limiting behavior of capacitors and inductors in DC

 acts like (while uncharged)

 (while fully charged)

(e.g. high pass filter question)

3) Virial theorem (and the quick way to get it)

m is reduced mass!

(To get levels for e.g. positronium, same formula but use reduced mass for that system)

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$$F(r) = Ar^{+n} \Rightarrow V = \frac{A}{1+n}r^{1+n} = \frac{F(r)}{1+n}r$$

$$\frac{mv^2}{r} = F(r) \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}F(r)r$$

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4) The Bohr formula (or know how to get it quickly)

$$E = -\frac{Z^2 (ke^2)^2 m}{2\hbar^2 n^2}$$

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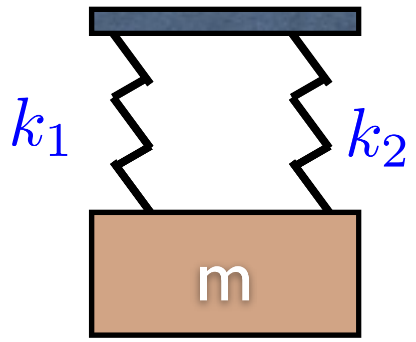
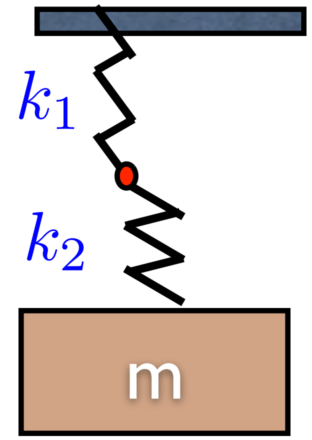
5) Combining masses, springs, capacitors, resistors





Can you find k_{equiv} ?

Frequency of oscillation?

Know reduced mass!

Quick Bohr (semi-classical) derivation

Electron traveling in a circle:

Quick Bohr (semi-classical) derivation

Electron traveling in a circle:

$$\frac{mv^2}{r} = \frac{k(Ze)(e)}{r^2}$$

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Put together to find r (Bohr radius!)

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$$\frac{1}{r} = \frac{kZe^2}{mv^2r^2} = \frac{kZme^2}{(mvr)^2} = \frac{kZme^2}{n^2\hbar^2}$$

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Potential energy:

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Potential energy:

$$PE = -\frac{kZe^2}{r} = -\frac{k^2Z^2me^4}{n^2\hbar^2}$$

Quick Bohr (semi-classical) derivation

Electron traveling in a circle:

$$\frac{mv^2}{r} = \frac{k(Ze)(e)}{r^2}$$

Angular momentum is quantized:

$$L = pr = mvr = n\hbar$$

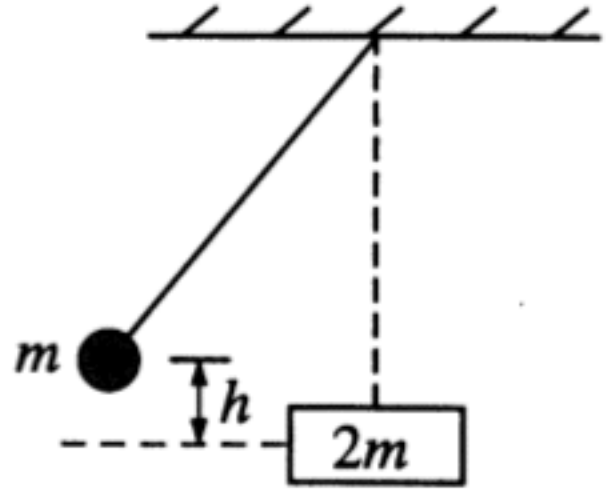
Put together to find r (Bohr radius!)

$$\frac{1}{r} = \frac{kZe^2}{mv^2r^2} = \frac{kZme^2}{(mvr)^2} = \frac{kZme^2}{n^2\hbar^2}$$

Potential energy:

$$PE = -\frac{kZe^2}{r} = -\frac{k^2Z^2me^4}{n^2\hbar^2}$$

Virial thm: $< E >= - < PE > /2$



7. As shown above, a ball of mass m, suspended on the end of a wire, is released from height h and collides elastically, when it is at its lowest point, with a block of mass $2m$ at rest on a frictionless surface. After the collision, the ball rises to a final height equal to

(A) 1/9 h
(B) 1/8 h
(C) 1/3 h
(D) 1/2 h
(E) 2/3 h

22

Apply elastic collision equations!

66. A sample of radioactive nuclei of a certain element can decay only by γ-emission and β-emission. If the half-life for γ-emission is 24 minutes and that for β-emission is 36 minutes, the half-life for the sample is

(A) 30 minutes
(B) 24 minutes
(C) 20.8 minutes
(D) 14.4 minutes
(E) 6 minutes

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$$t_{1/2} \stackrel{?}{=} t_\gamma + t_\beta \quad \text{or} \quad \frac{1}{t_{1/2}} \stackrel{?}{=} \frac{1}{t_\gamma} + \frac{1}{t_\beta}$$

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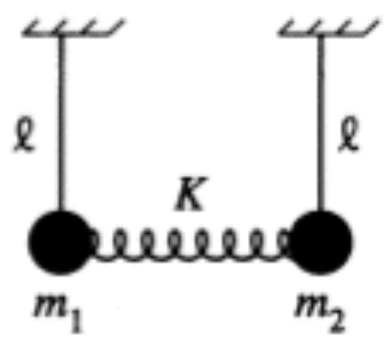
$$\frac{1}{t_{1/2}} = \frac{1}{t_\gamma} + \frac{1}{t_\beta} = \frac{1}{24} + \frac{1}{36}$$

$$= \frac{1}{6}\left(\frac{1}{4} + \frac{1}{6}\right)$$

$$= \frac{1}{6}\frac{10}{24}$$

$$\Rightarrow t_{1/2} = \frac{24 \times 6}{10} = \frac{144}{10} = 14.4 \text{ min}$$



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical lengths ℓ, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of this system?

(A) $\sqrt{g/\ell}$

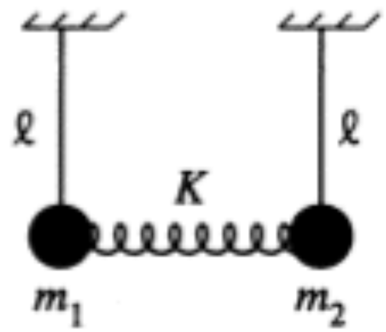
(B) $\sqrt{\dfrac{K}{m_1 + m_2}}$

(C) $\sqrt{\dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(D) $\sqrt{\dfrac{g}{\ell} + \dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(E) $\sqrt{\dfrac{2g}{\ell} + \dfrac{K}{m_1 + m_2}}$

20



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$$f_{\text{high}} \xrightarrow{K \text{ large}} ??$$

$$f_{\text{high}} \xrightarrow{K \text{ small}} ??$$

Reduced mass

20

What is the emission energy from a photon going from n = 3 to n = 1 in *positronium* (one electron and one positron orbiting one another)?

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Rule for Hydrogen like atoms: $E_n = \frac{-13.6 \text{ eV}}{n^2} Z^2$

But 13.6 is proportional to the *reduced* mass $m = m_{\text{electron}}$ in Hydrogen

In positronium $m = m_e/2$, so we have to halve the 13.6

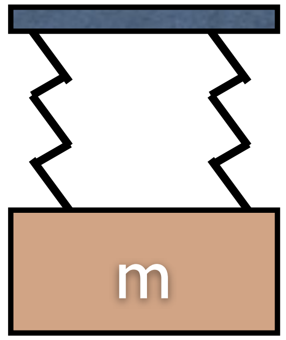
$$\boxed{E_n = -\frac{6.8 \text{ eV}}{n^2}}, \qquad \text{(positronium energy levels)}$$

$$E_{\text{photon}} = E_3 - E_1 = 6.8 \text{ eV} \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = 6.8 \text{ eV} \times \frac{8}{9} \approx 6 \text{ eV}$$

Situation 1)   Situation 2)

Two different ways of connecting a mass *m* to two *identical springs* with spring constant k are shown above. If we denote the frequency of oscillation in situation 1 by f_1 and the frequency of oscillation in situation 2 by f_2 then f_1/f_2 is:

a) 4
b) 2
c) 1/2
d) 1/4
e) depends on m and / or k



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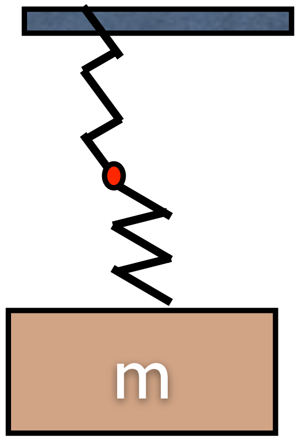

Situation 2)

Two different ways of connecting a mass *m* to two *identical springs* with spring constant k are shown above. If we denote the frequency of oscillation in situation 1 by f_1 and the frequency of oscillation in situation 2 by f_2 then f_1/ f_2 is:

a) 4
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(Hint: can pretend k_1 and k_2 are not the same to take limits to determine formula for $k_{\rm eff}$)

$$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{k_{\rm eff,1}/m}{k_{\rm eff,2}/m}} = \sqrt{\frac{k_{\rm eff,1}}{k_{\rm eff,2}}} = \sqrt{\frac{2k}{k/2}} = 2$$

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

A particle sits in a periodic potential

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What is its oscillation frequency about the minimum?

Let y be the distance from the minimum. Expanding about the minimum we have:

$$V(y) = V_{\min} + 0y + \frac{1}{2}\frac{d^2V}{dy^2}|_{\min} y^2 + \ldots$$

0 (because min)

Just a number, not a function

Force is

$$F = -\frac{dV}{dy} = -\frac{d^2V}{dy^2}|_{\min} y + \ldots$$

SHM with "spring constant" $k = d^2V/dy^2$ evaluated at min!

spring constant $= -dk^2\sin(kx) = +dk^2$ evaluated at min

$$f = 2\pi\sqrt{\frac{\text{spring const.}}{m}} = 2\pi\sqrt{\frac{dk^2}{m}}$$

6) Making problems look like a harmonic oscillator

$$\omega^2 = \frac{(d^2V/dx^2)|_{\min}}{m}$$

7) Remember spectroscopic notation (ugh)

$$^{2s+1}(\text{orbital angular momentum symbol})_j$$

and the selection rules for an electric dipole

8) Know the *pattern* of spherical harmonics

Y_ℓ^m

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}} \quad (\ell = 0)$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta \quad (\ell = 1)$$

$$Y_1^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{-i\varphi}$$

$$Y_2^0(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1) \quad (\ell = 2)$$

$$Y_2^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{i\varphi}$$

$$Y_2^2(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{2i\varphi}$$

Too detailed ...... !

(But if you can remember these, congratulations)

8) Know the *pattern* of spherical harmonics

$$Y_\ell^m$$

m – magnetic quantum number $(-\ell, -\ell+1, \ldots, \ell)$

ℓ – orbital quantum number $(0, 1, 2, \ldots)$

Y_ℓ^m contains φ dependence of the form $e^{im\phi}$

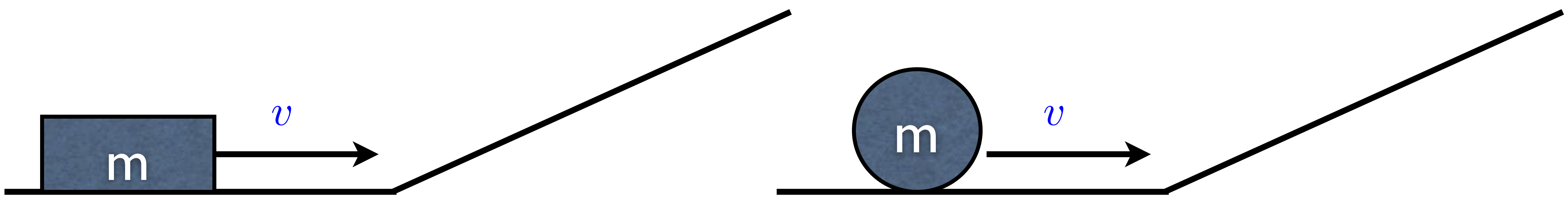
Y_ℓ^m contains ℓ dependence of the form $\sin^\ell \theta, \quad \sin^{\ell-1}\theta \cos\theta, \ldots$

(i.e. can write as ℓ sines or cosines mulitpled, or as $\sin(\ell\theta)$, $\cos(\ell\theta)$.)

Compare these rules to the spherical harmonics listed one slide ago.

Random mechanics problem:

A ball and a block of mass m are moving at the same speed *v*. When they hit the ramp they both travel up it. The block slides up with (approximately) no friction, the ball experiences just enough friction to roll without slipping. Which goes higher?



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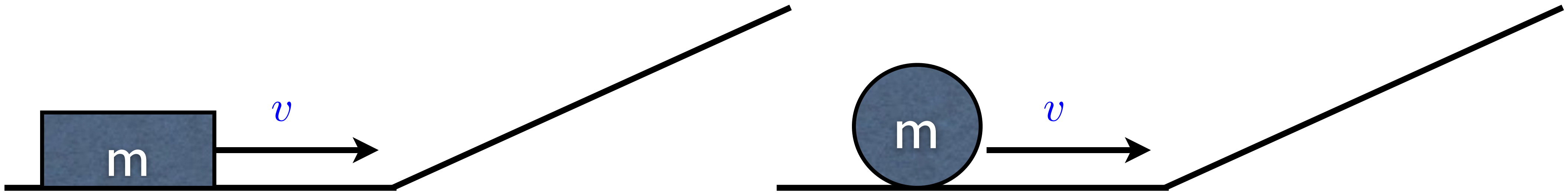

a) the ball goes higher
b) the black goes higher
c) they go the same height
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The ball has both translational kinetic energy (equal to that of the block) *and* rotational kinetic energy. Therefore

$$KE_{ball,initial} > KE_{block,initial}$$

The ball converts all this energy into potential energy, and therefore goes higher.