The Northern California Physics GRE Bootcamp

Held at UC Davis, August 11-12, 2018

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Big tips and tricks

* Multiple passes through the exam

* Dimensional analysis (which answers make sense?)
Other hint -- look at exponentials, sines, cosines, ...

* Expansions, in particular $(1+x)^n = 1 + n x + \ldots\ldots\ldots$

* Limiting cases (e.g. make parameters go to 0 or infinity)

* Special cases (e.g. looking at circles)

* Powers of ten estimation

* Know scales of things [wavelength / freq of visible light, binding energies of nuclei, mass ratios of common particles (up to muon, pion), ..., mass of stars, mass of galaxies, ...]

Okay to specialize on scales

Big tips and tricks -- material

* Know your "first year" general physics really well
  - Newtonian mechanics in particular

* Know your "modern physics" general physics really well
  (usually Sophomore class)

* Worth going through
  Griffiths: Intro to electromagnetism
  Griffiths: Intro to quantum mechanics
  (Concentrate on harmonic osc, infinite square well, spin systems, expectation values)
  Schroeder: Thermal physics

* Look at the archive of monthly problems in *The Physics Teacher* (if you have access to a university library)

45. During a hurricane, a 1,200 Hz warning siren on the town hall sounds. The wind is blowing at 55 m/s in a direction from the siren toward a person 1 km away. With what frequency does the sound wave reach the person? (The speed of sound in air is 330 m/s.)

(A) 1,000 Hz
(B) 1,030 Hz
(C) 1,200 Hz
(D) 1,400 Hz
(E) 1,440 Hz

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Evaluate whether question is "special" or "first year"

96. Which of the following ions CANNOT be used as a dopant in germanium to make an *n*-type semiconductor?

(A) As
(B) P
(C) Sb
(D) B
(E) N

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Specific tips and tricks -- material

Bare bones thermal:
Know the definition of partition function, probability of a state, how to find an expectation value

$$Z = \sum_{\text{state i}} \exp(-\beta E_i) = \sum_{\text{energies j}} g_j \exp(-\beta E_j)$$

$$\text{Prob(State i)} = \frac{1}{Z} e^{-\beta E_i}$$

$$\langle X \rangle = \sum_{\text{state i}} X_i P(X_i) = \frac{1}{Z} \sum_{\text{state i}} X_i \exp(-\beta E_i)$$

Bare bones quantum:
Know the "modern physics" course, know how to find probability of a state, know how to find expectation values, know the special systems, know spin-addition rules

special systems: *particle in box, harmonic oscillator, two spin 1/2 particles*

Specific tips and tricks -- material

Bare bones (advanced) classical:
Know ***how to find*** *the Hamiltonian, Hamilton's equations, Lagrangian, and the Euler Lagrange equations for a particle in a gravitational field, charged particle in a uniform electric field, and a pendulum.*

Don't just memorize the results, if you can do these three systems you will be aware of the pattern.

Remember: GRE questions are typically short -- cannot get you to do any crazy calculations!

$$C = 3kN_A\left(\frac{hv}{kT}\right)^2 \frac{e^{hv/kT}}{(e^{hv/kT} - 1)^2}$$

65. Einstein's formula for the molar heat capacity C of solids is given above. At high temperatures, C approaches which of the following?

(A) 0

(B) $3kN_A\left(\frac{hv}{kT}\right)$

(C) $3kN_A hv$

(D) $3kN_A$

(E) $N_A hv$

Dimensions here

$$C = 3kN_A\left(\frac{hv}{kT}\right)^2 \frac{e^{hv/kT}}{(e^{hv/kT}-1)^2}$$

This quantity is dimensionless

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~~(C) $3kN_A hv$~~

(D) $3kN_A$

~~(E) $N_A hv$~~

Call $x = \frac{hv}{kT}$

$$\begin{aligned}
C &= 3kN_A x^2 \frac{e^x}{(e^x-1)^2} \\
&= 3kN_a x^2 \left[\frac{1+x+\ldots}{((1+x+\ldots-1)^2}\right] \\
&= 3kN_a x^2 \left[\frac{1}{x^2}+\ldots\right] \\
&= 3kN_a + \ldots
\end{aligned}$$

A distant galaxy is observed to have its H-beta line shifted to a wavelength of 480nm from its laboratory value of 434nm. Which is the best approximation to the velocity of the galaxy? (Note: 480/434 ~ 1.1)

a) 0.01c
b) 0.05c
c) 0.1c
d) 0.32c
e) 0.5c

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$$\lambda_{\text{obs}} = \lambda_{\text{emit}} \sqrt{\frac{c+v}{c-v}}$$

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \sqrt{\frac{1+(v/c)}{1-(v/c)}} \approx \sqrt{(1+(v/c))^2}$$

$$v \approx \left(\frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1\right) c$$

14. An 8-centimeter-diameter by 8-centimeter-long NaI(Tl) detector detects gamma rays of a specific energy from a point source of radioactivity. When the source is placed just next to the detector at the center of the circular face, 50 percent of all emitted gamma rays at that energy are detected. If the detector is moved to 1 meter away, the fraction of detected gamma rays drops to

(A) 10^{-4}

(B) 2×10^{-4}

(C) 4×10^{-4}

(D) $8\pi \times 10^{-4}$

(E) $16\pi \times 10^{-4}$

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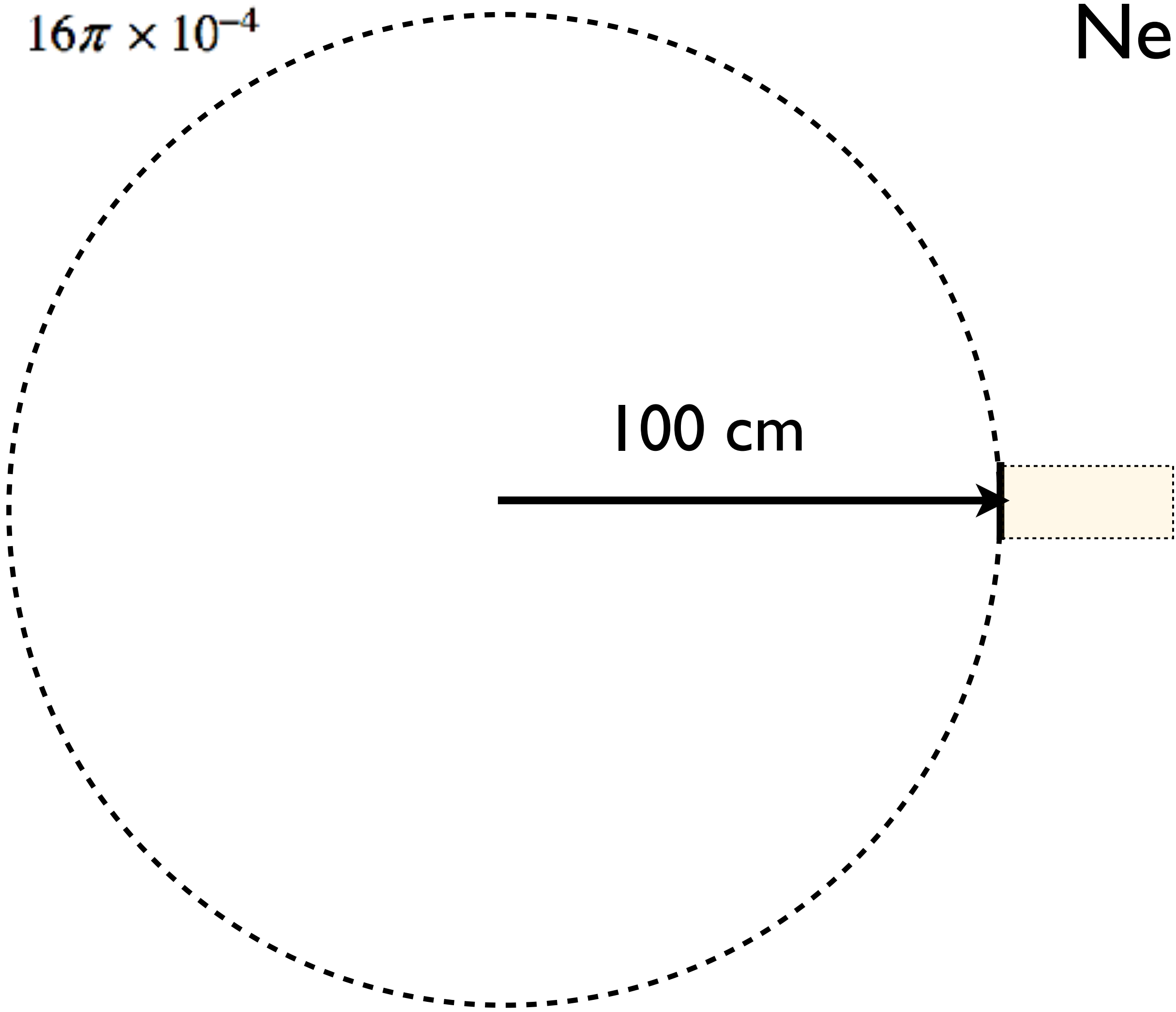

Need fraction of area covered by sensor

$$A_{\text{sensor}} = \pi(4\text{ cm})^2 = 16\pi\text{ cm}^2$$

$$A_{\text{sphere}} = 4\pi(100\text{ cm}^2) = 4\pi \times 10^4\text{ cm}^2$$

$$\frac{A_{\text{sensor}}}{A_{\text{sphere}}} = \frac{16\pi}{4\pi \times 10^4} = 4 \times 10^{-4}$$

2. The coefficient of static friction between a small coin and the surface of a turntable is 0.30. The turntable rotates at 33.3 revolutions per minute. What is the maximum distance from the center of the turntable at which the coin will not slide?

(A) 0.024 m
(B) 0.048 m
(C) 0.121 m
(D) 0.242 m
(E) 0.484 m

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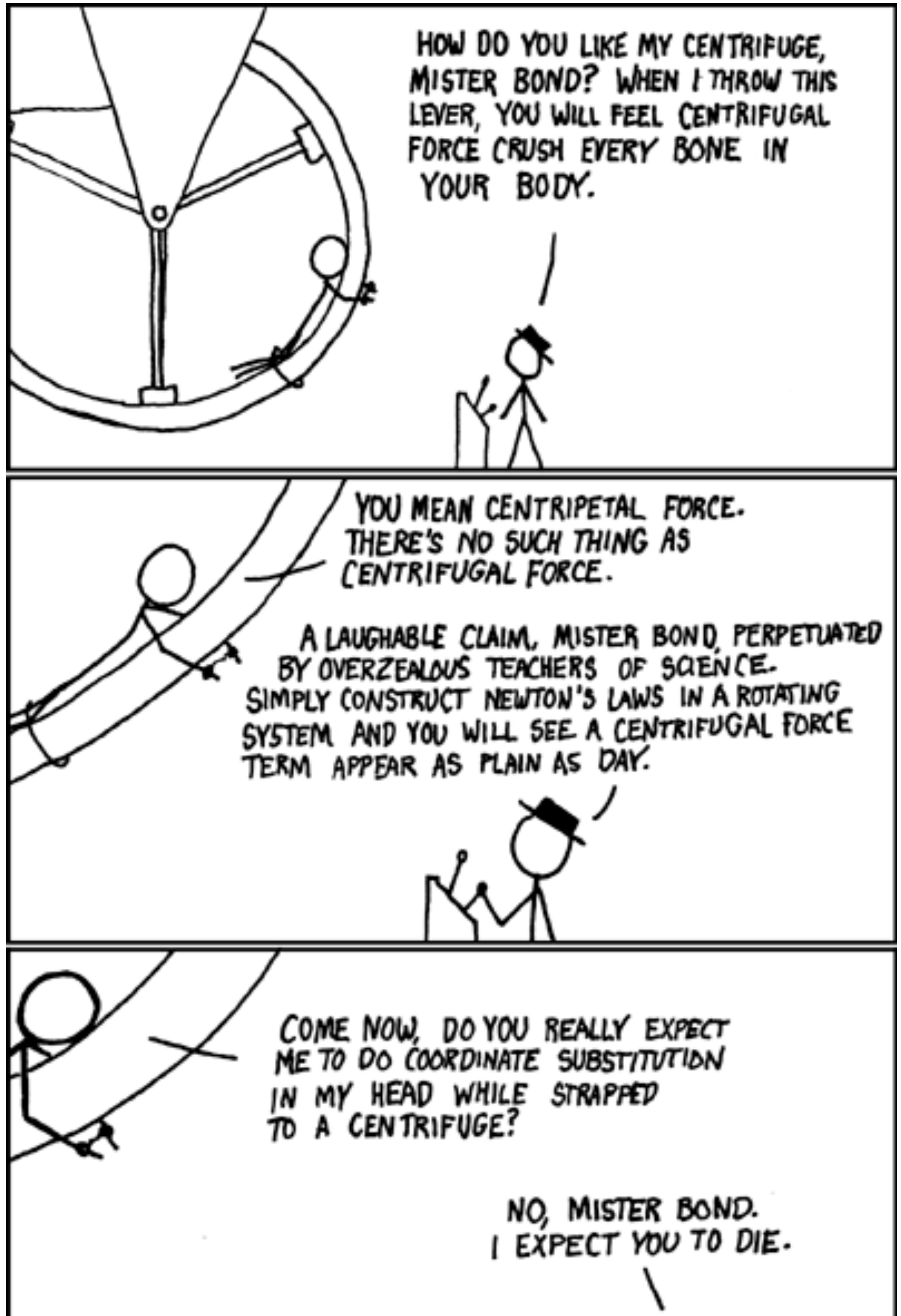
max friction = "centrifugal" force [http://xkcd.com/123/]

$\mu N = \mu m g = m\omega^2 r$ **so** $r = \dfrac{\mu g}{\omega^2}$

$$\omega = \frac{2\pi}{T} \text{ (don't use!)}$$

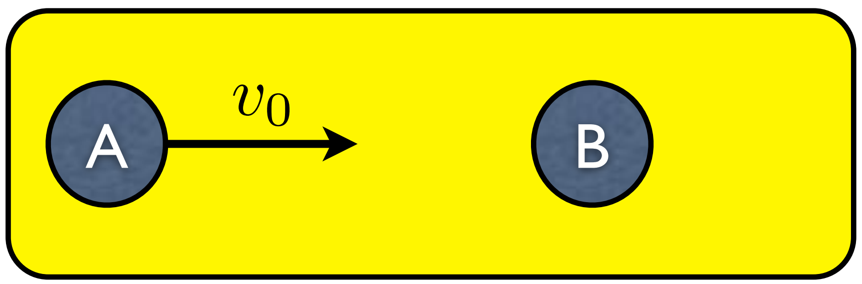
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \times 33.3}{60 \text{ s}} \sim \pi \text{ s}^{-1} \qquad \Rightarrow \omega^2 \sim 10 \text{ s}^{-2}$$ therefore

$$r \sim \mu \text{ m} \sim 0.3 \text{ m}$$

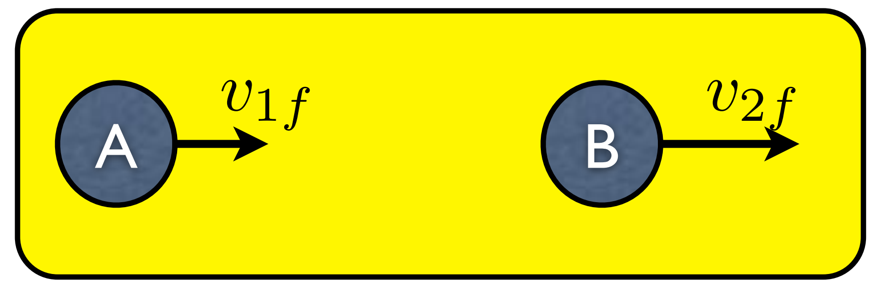

HOW DO YOU LIKE MY CENTRIFUGE, MISTER BOND? WHEN I THROW THIS LEVER, YOU WILL FEEL CENTRIFUGAL FORCE CRUSH EVERY BONE IN YOUR BODY.
YOU MEAN CENTRIPETAL FORCE. THERE'S NO SUCH THING AS CENTRIFUGAL FORCE.
A LAUGHABLE CLAIM, MISTER BOND, PERPETUATED BY OVERZEALOUS TEACHERS OF SCIENCE. SIMPLY CONSTRUCT NEWTON'S LAWS IN A ROTATING SYSTEM AND YOU WILL SEE A CENTRIFUGAL FORCE TERM APPEAR AS PLAIN AS DAY.
COME NOW, DO YOU REALLY EXPECT ME TO DO COORDINATE SUBSTITUTION IN MY HEAD WHILE STRAPPED TO A CENTRIFUGE?
NO, MISTER BOND. I EXPECT YOU TO DIE.

Things to know (they always seem to come up)

1) Elastic collision formula



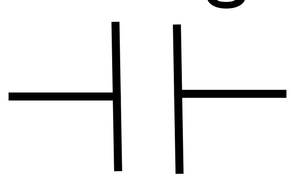
Before collision



After collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_0$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_0$$

2) The limiting behavior of capacitors and inductors in DC

 acts like (while uncharged)

 (while fully charged)

(e.g. high pass filter question)

3) Virial theorem (and the quick way to get it)

$$F(r) = Ar^{+n} \Rightarrow V = \frac{A}{1+n} r^{1+n} = \frac{F(r)}{1+n} r$$

$$\frac{mv^2}{r} = F(r) \Rightarrow \frac{1}{2} mv^2 = \frac{1}{2} F(r) r$$

$$\boxed{\langle KE \rangle = \frac{1+n}{2} \langle V \rangle}$$

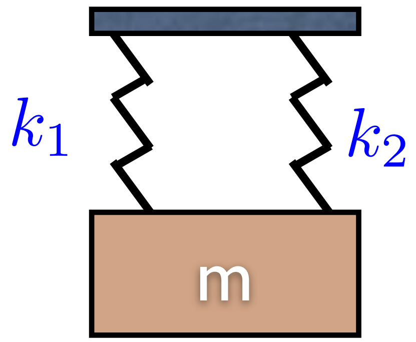
4) The Bohr formula (or know how to get it quickly)

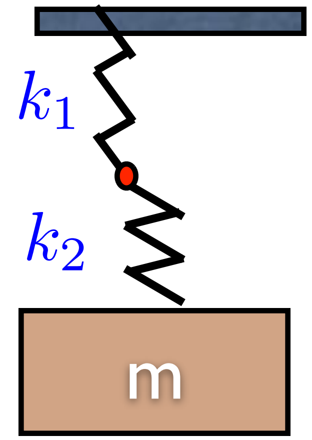
$$\boxed{E = -\frac{Z^2 (ke^2)^2 m}{2\hbar^2 n^2}}$$

m is reduced mass!

(To get levels for e.g. positronium, same formula but use reduced mass for that system)

5) Combining masses, springs, capacitors, resistors





Can you find k_{equiv} ?

Frequency of oscillation?

Know reduced mass!

Quick Bohr (semi-classical) derivation

Electron traveling in a circle:

$$\frac{mv^2}{r} = \frac{k(Ze)(e)}{r^2}$$

Angular momentum is quantized:

$$L = pr = mvr = n\hbar$$

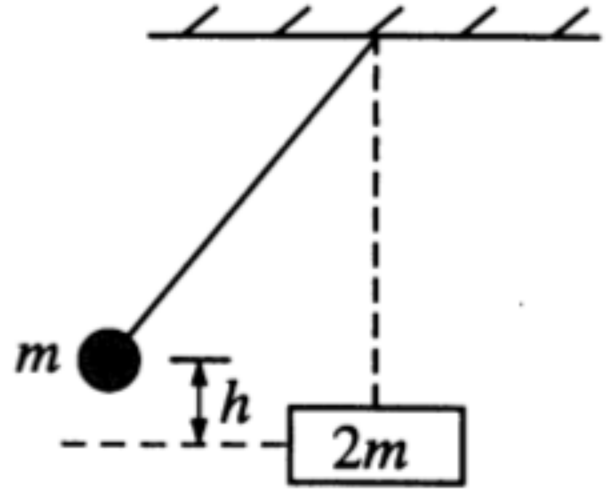
Put together to find r (Bohr radius!)

$$\frac{1}{r} = \frac{kZe^2}{mv^2r^2} = \frac{kZme^2}{(mvr)^2} = \frac{kZme^2}{n^2\hbar^2}$$

Potential energy:

$$PE = -\frac{kZe^2}{r} = -\frac{k^2Z^2me^4}{n^2\hbar^2}$$

Virial thm: $< E >= - < PE > /2$



7. As shown above, a ball of mass m, suspended on the end of a wire, is released from height h and collides elastically, when it is at its lowest point, with a block of mass $2m$ at rest on a frictionless surface. After the collision, the ball rises to a final height equal to

(A) 1/9 h
(B) 1/8 h
(C) 1/3 h
(D) 1/2 h
(E) 2/3 h

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Apply elastic collision equations!

66. A sample of radioactive nuclei of a certain element can decay only by γ-emission and β-emission. If the half-life for γ-emission is 24 minutes and that for β-emission is 36 minutes, the half-life for the sample is

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(B) 24 minutes
(C) 20.8 minutes
(D) 14.4 minutes
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$$t_{1/2} \stackrel{?}{=} t_\gamma + t_\beta \quad \text{or} \quad \frac{1}{t_{1/2}} \stackrel{?}{=} \frac{1}{t_\gamma} + \frac{1}{t_\beta}$$

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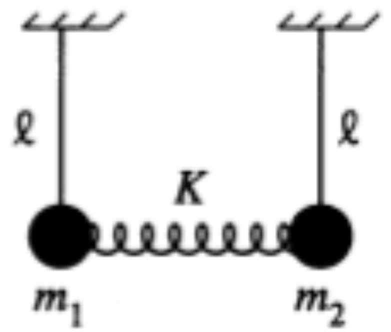
$$\cancel{t_{1/2} \stackrel{?}{=} t_\gamma + t_\beta} \quad \text{or} \quad \frac{1}{t_{1/2}} \stackrel{?}{=} \frac{1}{t_\gamma} + \frac{1}{t_\beta}$$

$$\begin{aligned} \frac{1}{t_{1/2}} = \frac{1}{t_\gamma} + \frac{1}{t_\beta} &= \frac{1}{24} + \frac{1}{36} \\ &= \frac{1}{6}\left(\frac{1}{4} + \frac{1}{6}\right) \\ &= \frac{1}{6}\frac{10}{24} \end{aligned}$$

$$\Rightarrow t_{1/2} = \frac{24 \times 6}{10} = \frac{144}{10} = 14.4 \text{ min}$$



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical lengths ℓ, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of this system?

(A) $\sqrt{g/\ell}$

(B) $\sqrt{\dfrac{K}{m_1 + m_2}}$

(C) $\sqrt{\dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(D) $\sqrt{\dfrac{g}{\ell} + \dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(E) $\sqrt{\dfrac{2g}{\ell} + \dfrac{K}{m_1 + m_2}}$

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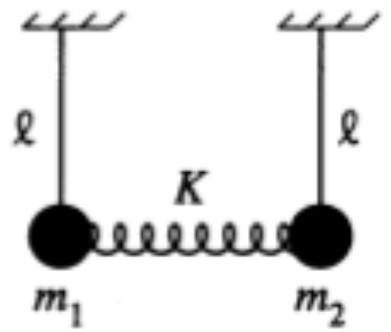

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20

$$f_{\text{high}} \xrightarrow{K \text{ large}} ??$$

$$f_{\text{high}} \xrightarrow{K \text{ small}} ??$$

Reduced mass

What is the emission energy from a photon going from n = 3 to n = 1 in *positronium* (one electron and one positron orbiting one another)?

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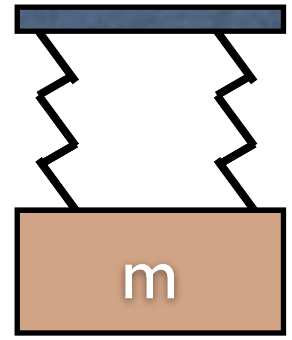
Rule for Hydrogen like atoms: $E_n = \frac{-13.6 \text{ eV}}{n^2} Z^2$

But 13.6 is proportional to the *reduced* mass $m = m_{\text{electron}}$ in Hydrogen

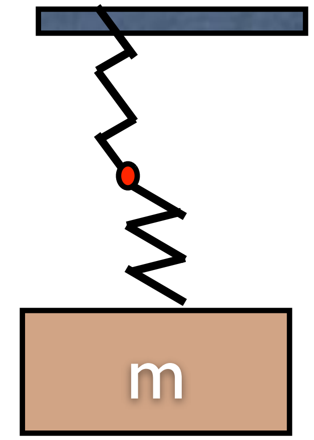
In positronium $m = m_e/2$, so we have to halve the 13.6

$$\boxed{E_n = -\frac{6.8 \text{ eV}}{n^2}}, \qquad \text{(positronium energy levels)}$$

$$E_{\text{photon}} = E_3 - E_1 = 6.8 \text{ eV} \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = 6.8 \text{ eV} \times \frac{8}{9} \approx 6 \text{ eV}$$



Situation 1)



Situation 2)

Two different ways of connecting a mass *m* to two *identical springs* with spring constant k are shown above. If we denote the frequency of oscillation in situation 1 by f_1 and the frequency of oscillation in situation 2 by f_2 then f_1/ f_2 is:

a) 4
b) 2
c) 1/2
d) 1/4
e) depends on m and / or k

Situation 1)   Situation 2)

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(Hint: can pretend k_1 and k_2 are not the same to take limits to determine formula for k_{eff})

$$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{k_{\text{eff},1}/m}{k_{\text{eff},2}/m}} = \sqrt{\frac{k_{\text{eff},1}}{k_{\text{eff},2}}} = \sqrt{\frac{2k}{k/2}} = 2$$

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

Let y be the distance from the minimum. Expanding about the minimum we have:

$$V(y) = V_{\rm min} + \boxed{0y} + \frac{1}{2}\boxed{\frac{d^2V}{dy^2}|_{\rm min}} y^2 + \ldots$$

0 (because min)

Just a number, not a function

Force is

$$F = -\frac{dV}{dy} = -\frac{d^2V}{dy^2}|_{\rm min} y + \ldots$$

SHM with "spring constant" $k = d^2V/dy^2$ evaluated at min!

spring constant $= -dk^2 \sin(kx) = +dk^2$ evaluated at min

$$f = 2\pi\sqrt{\frac{\text{spring const.}}{m}} = 2\pi\sqrt{\frac{dk^2}{m}}$$

6) Making problems look like a harmonic oscillator

$$\omega^2 = \frac{(d^2V/dx^2)|_{\min}}{m}$$

7) Remember spectroscopic notation (ugh)

$$^{2s+1}(\text{orbital angular momentum symbol})_j$$

and the selection rules for an electric dipole

8) Know the *pattern* of spherical harmonics

Y_ℓ^m

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$(\ell = 0)$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta$$

$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

$(\ell = 1)$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{-i\varphi}$$

$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1)$$

$$Y_2^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{i\varphi}$$

$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{2i\varphi}$$

$(\ell = 2)$

Too detailed ...... !

(But if you can remember these, congratulations)

8) Know the *pattern* of spherical harmonics

$$Y_\ell^m$$

m – magnetic quantum number $(-\ell, -\ell+1, \ldots, \ell)$

ℓ – orbital quantum number $(0, 1, 2, \ldots)$

Y_ℓ^m contains φ dependence of the form $e^{im\phi}$

Y_ℓ^m contains ℓ dependence of the form $\sin^\ell\theta, \quad \sin^{\ell-1}\theta\cos\theta, \ldots$

(i.e. can write as ℓ sines or cosines mulitpled, or as $\sin(\ell\theta)$, $\cos(\ell\theta)$.)

Compare these rules to the spherical harmonics listed one slide ago.